Non-Probabilistic Betting

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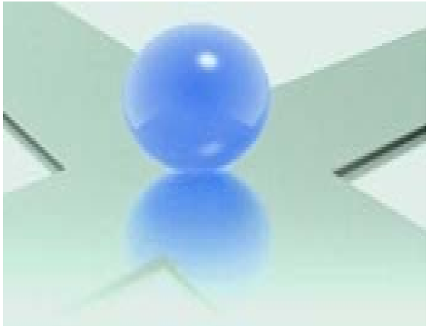

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Contents

§ Sources:

- **Yakov Ben-Haim, 2011,**

Robustness and Locke's wingless gentleman,

http://decisions-and-info-gaps.blogspot.com /2011/09/robustness-and-lockes-wingless.html

- **Yakov Ben-Haim, 2011,**

Squirrels and Stock Brokers, Or:

Dilemmas of Decision Making,

  - **http://decisions-and-info-gaps.blogspot.com /2011/10/squirrels-and-stock-brokers-or.html**
  - **\papers\essays\Decis-Dilem-Unc\lgsq01.pdf**

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§ Main ideas:

- **Decisions aim to achieve goals.**

  What are good decision methods?

- **Innovation dilemma:**

  better option more uncertain.

- **Robustness:**
  - **Relevance for innovation dilemma.**
  - **Relation to betting and probability.**

- **Does robustness assume probability?**

1 *Robustness and Locke's Wingless Gentleman*

§ What is a bet?

- **OED:** “To stake or wager (a sum of money, etc.) in support of an affirmation or on the issue of a forecast.”
- Examples:
  - “Iohn a Gaunt loued him well, and betted much money on his head.” (Shakespeare, 1600).
  - “For a long while it was an euen bet . . . Whether proud Warwick, or the Queene should win.” (Drayton, 1627)

§ **Even bet: 50-50 chance, equal prob.**

§ **Betting not always chancey:**

- **OED "You bet" or "You bet you" mean "be assured, certainly".**
- **Examples:**
  - **" 'Can you handle this outfit?' 'You bet,' said the scout." (Sayers)**
  - **" 'I'll get you there on time'—and you bet you he did, too." (Twain)**

§ "Bet": both certainty and uncertainty.

- **Twain's 'you bet you' states certainty.**
- **Drayton's 'even bet': no idea who'll win.**

§ Betting: a dialectic.

- **Dialectic:**

  Opposites combine into synthesis.
- **Dialectic of uncertainty:**

  Doubt & determination form decisiveness.

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- Dialectic:

  Opposites combine into synthesis.

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§ Locke's wingless gentleman:

> If we will disbelieve everything, because we cannot certainly know all things; we shall do muchwhat as wisely as he, who would not use his legs, but sit still and perish, because he had no wings to fly. (*An Essay Concerning Human Understanding*, 1706, I.i.5)

The consequence of unabated doubt—paralysis—induces doubt's opposite: decisiveness.

§ How to bet without probability?

§ Robust means:

- **Strong, healthy.**
- **Not easily broken.**
- **Statistical test:**

  Approximately correct even if data or assumptions err.

§ Decision is robust if

outcome OK despite error.

§ Is non-probabilistic robustness a good probabilistic bet?

2 Squirrels, Stock-Brokers and Their Dilemmas

§ Decision problems:

- **Squirrels nibbling acorns.**
  - **These acorns are okay.**
  - **Distant oaks look better.**
- **Stock broker portfolio:**
  - **Current portfolio okay.**
  - **Portfolio with startup looks better.**

§ Decision problems:

- **Squirrels nibbling acorns.**
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§ Traits of decision problems:

- **Critical needs must be met.**
- **Current situation may or may not be adequate.**
- **One option looks better but is more uncertain. (Innovation dilemma)**

§ Solution strategies: extreme cases.

- **Status quo certainly inadequate.**
  - **Squirrel starves: must try new oaks.**
  - **Broker goes broke: must try startup.**
- **Status quo certainly adequate.**
  - **Squirrel survives: why risk new oaks?**
  - **Broker beats competition: why change?**

§ **Solution strategies: extreme cases.**

- **Status quo certainly inadequate.**
  - **Squirrel starves: must try new oaks.**
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- **Status quo certainly adequate.**
  - **Squirrel survives: why risk new oaks?**
  - **Broker beats competition: why change?**

§ **General conclusion:**

- **Right answer depends on what you need.**
  Depends on what is "adequate".
- **No universal rule, like:**
  - **'Always try to improve.'**
  - **'If it's working, don't fix it.'**
- **Satisficing decision strategy.**
- **Preference reversal:**
  Choice changes as need changes.

§ Preference reversal:

Choice changes as need changes.

§ Innovation dilemma:

- **Choose between 2 options.**
- **1 option seems better but is more uncertain.**
- **Preference reversal.**

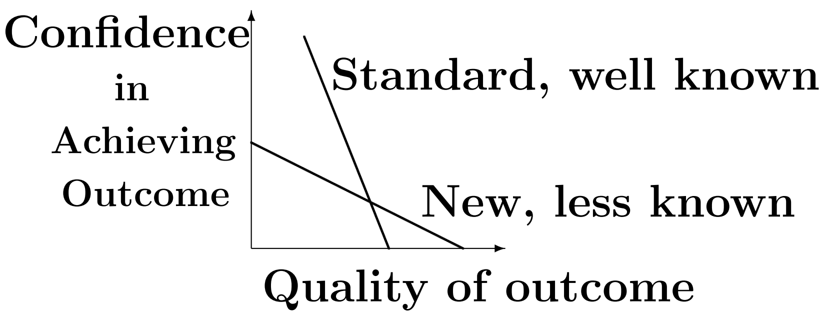

Figure 1: **Innovation dilemma.**

§ Other strategy: Best-model optimization

- **Obtain info, understanding, models.**
- **Find action w/ best predicted outcome.**

§

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§ Best-model optimization:

- **Usually unique choice.**
- **Moral imperative: do your best!**
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- **Catch: Uncertainty.**

  Best info may be very wrong.

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§ Stock broker's answer:

"I have probabilistic asset pricing model."

§ Response:

- **Best prob model optim is special case of best model optimization.**
- **Probabilistic models err.**

3 Robustness as a Proxy for Probability

§ Robustness is (often) a proxy for probability.

- **Robust decision uses no probab info.**
- **More robust decision (often) more likely to succeed.**
- **Examples:**[1]
  - **Biological evolution: foraging.**
  - **Finance m'kts: equity premium puzzle.**
  - **Human psych: maximizers have more, satisficers are happier.**

[1] See lecture: \lectures\decisions\lectures\pdox-choice\pdox-choice01.tex

§ Proxy property: simple special case.

- **Squirrel and stock broker examples.**
  Options: "Stay" or "Move".
- **First case:**
  - **Survival sure with "Stay".**
  - **Survival unsure with "Move".**
  - **Survival prob = 1 with "Stay".**
  - **"Stay" more robust and better bet regardless of "Move" probability.**

§ Proxy property: simple special case.

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  - **Survival unsure with "Move".**
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  - **"Stay" more robust and better bet regardless of "Move" probability.**
- **Second case:**
  - **Failure sure with "Stay".**
  - **Failure unsure with "Move".**
  - **Failure prob = 1 with "Stay".**
  - **"Move" more robust and better bet regardless of "Stay" probability.**
-

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  - **Failure unsure with "Move".**
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  - **"Move" more robust and better bet regardless of "Stay" probability.**
- **Robustness proxies for probability.**

4 *Is Robustness Probabilistic? 3 Questions*

§ Source:

- **Yakov Ben-Haim, 2011, Uncertainty, Probability and Robust Preferences \papers\Uncer-Prob\up03.tex**

[1] \lectures\talks\lib\is-rbs-prob-3qs01.tex 16.11.2011

§ Main points:

- **Uncertainty & probability are different.**
- **Reasoning about uncertainty does not require probability.**

§ **Main points:**

- **Uncertainty & probability are different.**
- **Reasoning about uncertainty does not require probability.**

§ **We ask 3 questions:**

Does a non-probabilistic robust preference between options:

- **Assume a uniform probability distribution?**
- **Assume *some* probability distribution?**
- **Make any probabilistic assumptions?**

4.1 *Two Views on Uncertainty and Probability*

§ Cogent reasoning about uncertainty without probability?

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§ Cogent reasoning about uncertainty without probability?

- **Answer obvious to most people.**
  Some say YES, some say NO.

§ **Probability is fundamental to uncertainty.**

§ **Keynes:**

> **"Part of our knowledge we obtain direct; and part by argument. The Theory of Probability is concerned with that part which we obtain by argument, and it treats of the different degrees in which the results so obtained are conclusive or inconclusive. . . .**
>
> **"The method of this treatise has been to regard subjective probability as fundamental and to treat all other relevant conceptions as derivative from this."**

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"The method of this treatise has been to regard subjective probability as fundamental and to treat all other relevant conceptions as derivative from this."

§ Among Carnap's "basic conceptions" is the contention that

"all inductive reasoning, in the wide sense of nondeductive or nondemonstrative reasoning, is reasoning in terms of probability."

§ **Probability not fundamental to uncertainty.**

§ **Knight distinguished between**

- **Probabilistic uncertainty: risk.**
- **Unmeasurable uncertainty:**
  "true uncertainty".

§

§ Probability not fundamental to uncertainty.

§ Knight distinguished between

- Probabilistic uncertainty: risk
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  "true uncertainty".

§ Wald wrote that

> "in most of the applications not even the existence of ... an a priori probability distribution [on the class of distribution functions] ... can be postulated, and in those few cases where the existence of an a priori probability distribution ... may be assumed this distribution is usually unknown."

§ Cogent reasoning about uncertainty without probability?

- **My answer: Yes.**
- **Some reasons apodictic.**
- **Some reasons contingent.**

4.2 *What are Non-Probabilistic Preferences?*

§ Models of uncertainty:

- **Measure theoretic:**
  probability, fuzzy, imprecise probability, generalized information theory.
- **Set theoretic (non-probabilistic):**
  worst-case, interval, ellipsoid, bounded-error, p-boxes, info-gap.

§ **Models of uncertainty:**

- **Measure theoretic:**
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§ **Preferences with set-theoretic uncertainty:**

- **Min-max.**
- **Satisficing.**
- **Robust satisficing.**
- **Windfalling.**
- **Opportune windfalling.**

§ Notation:

- **Two options:** B **and** C.
- **Evaluate their robustnesses.**
- B **is more robust than** C**:** $B \succ_{\mathrm{r}} C$**.**

4.3 *First Question*

§ Given robust preference algorithm, $\succ_{\mathrm{r}}$.

§ Question:

Does the justification of $\succ_{\mathrm{r}}$ need to assume a uniform probability distribution of the uncertain events?

§ **Given robust preference algorithm, $\succ_{\rm r}$.**

§ **Question:**

Does the justification of $\succ_{\rm r}$ need to assume a uniform probability distribution of the uncertain events?

§ **Why is it often assumed “Yes”?**

- **Ignorance is probabilistic:**
  - **Principle of indifference.**
  - **Principle of maximum entropy.**
-

§ Given robust preference algorithm, $\succ_{\rm r}$.

§ Question:

Does the justification of $\succ_{\rm r}$ need to assume a uniform probability distribution of the uncertain events?

§ Why is it often assumed "Yes"?

- **Ignorance is probabilistic:**
  - **Principle of indifference.**
  - **Principle of maximum entropy.**
- **Confusion with converse:**
  - **Larger set has greater robustness.**
  - **Uniform pdf $\Longrightarrow$ larger set more likely.**
  - **Hence $\succ_{\rm r}$ presumes uniform pdf.**

§ **Many pdf's could justify $\succ_{\mathrm{r}}$.**

Examples:

- **3-door problem.**
- **Lewis Carroll's 2-counter problem.**

§ **Thus uniform pdf not necessary to justify $\succ_{\mathrm{r}}$.**

4.4 *Second Question*

§ Question:

Does the justification of $\succ_{\rm r}$ need to assume *some* probability distribution of the uncertain events?

§ Notation:

- u **is underlying uncertain event.**
- $p(u)$ **is pdf for** u**.**
- S **is set of all** $p(u)$**'s.**
- $p_{\mathrm{T}}(u)$ **is true pdf.**
- $S_B \subset S$ **containing all** $p(u)$**'s for which** B **is more likely to succeed than** C**.**

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- $S_B \subset S$ **containing all** $p(u)$**'s for which** B **is more likely to succeed than** C**.**

§ Our question is:

Must we assume $p_{\mathrm{T}} \in S_B$ **to justify** $\succ_{\mathrm{r}}$ **?**

§ Must we assume $p_{\mathrm{T}} \in S_B$ to justify $\succ_{\mathrm{r}}$?

§ It is true:

$$p_{\mathrm{T}} \in S_B \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{1}$$

'$\Longrightarrow$' means 'justifies', 'warrants', 'motivates'.

§ **Must we assume $p_{\mathrm{T}} \in S_B$ to justify $\succ_{\mathrm{r}}$?**

§ **It is true:**

$$p_{\mathrm{T}} \in S_B \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{2}$$

§ **It is also true:**

$$\mathbf{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{3}$$

"Prob": judgment on space of pdf's $p(u)$.

§ **Thus (2) not necessary for $\succ_{\mathrm{r}}$.**

§ **Must we assume $p_{\mathrm{T}} \in S_B$ to justify $\succ_{\mathrm{r}}$?**

§ **It is true:**

$$p_{\mathrm{T}} \in S_B \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{4}$$

§ **It is also true:**

$$\mathbf{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{5}$$

"Prob": judgment on space of pdf's $p(u)$.

§ **It is also true:**

$$\mathbf{Prob}\left(\mathbf{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2}\right) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{6}$$

§ **Thus neither (4) nor (5) necessary for $\succ_{\mathrm{r}}$.**

§ **Must we assume $p_{\rm T} \in S_B$ to justify $\succ_{\rm r}$?**

§ **It is true:**

$$p_{\rm T} \in S_B \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{7}$$

§ **It is also true:**

$$\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{8}$$

"Prob": judgment on space of pdf's $p(u)$.

§ **It is also true:**

$$\textbf{Prob}\left(\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2}\right) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{9}$$

§ **It is also true:**

$$\textbf{Prob}\left(\ldots\left[\textbf{Prob}\left(\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2}\right) > \frac{1}{2}\right]\right) > \frac{1}{2} \\ \Longrightarrow \quad B \succ_{\rm r} C \tag{10}$$

§ **Thus (7)–(9) not necessary for $\succ_{\rm r}$.**

§ **Must we assume $p_{\rm T} \in S_B$ to justify $\succ_{\rm r}$?**

§ **It is true:**

$$p_{\rm T} \in S_B \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{11}$$

§ **It is also true:**

$$\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{12}$$

"Prob": judgment on space of pdf's $p(u)$.

§ **It is also true:**

$$\textbf{Prob}\left(\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2}\right) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{13}$$

§ **It is also true:**

$$\textbf{Prob}\left(\ldots\left[\textbf{Prob}\left(\textbf{Prob}(p_{\rm T} \in S_B) > \frac{1}{2}\right) > \frac{1}{2}\right]\right) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\rm r} C \tag{14}$$

§ **Conclusions:**

- **Need not assume** $p_{\rm T} \in S_B$ **to justify** $\succ_{\rm r}$ **.**
- ∞ **of prob judgments would justify** $\succ_{\rm r}$ **.**

4.5 *Third Question*

§ Question:

Is at least one belief among:

$$\mathbf{Prob}\left(\ldots\left[\mathbf{Prob}\left(\mathbf{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2}\right) > \frac{1}{2}\right]\right) > \frac{1}{2}$$

$$\Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{15}$$

necessary to justify $\succ_{\mathrm{r}}$ **?**

§ Question:

Is at least one belief among:

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necessary to justify $\succ_{\mathrm{r}}$ **?**

§ Recall Locke's wingless gentleman:

> **If we will disbelieve everything, because we cannot certainly know all things; we shall do muchwhat as wisely as he, who would not use his legs, but sit still and perish, because he had no wings to fly. (*An Essay Concerning Human Understanding,* 1706, I.i.5)**

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$$\Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{17}$$

necessary to justify $\succ_{\mathrm{r}}$ **?**

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§ Recall dialectic of uncertainty:

Doubt & determination form decisiveness.

§ Pragmatic arguments for $\succ_{\mathrm{r}}$:

- **Avoid epistemic paralysis (Locke's guy).**
- **Avoid cost of inaction.**
- **Dialectic of uncertainty:**

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§ Pragmatic arguments for $\succ_{\mathrm{r}}$:

- **Avoid epistemic paralysis (Locke's guy).**
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Doubt & determination form decisiveness.

§ Are these good arguments?

Are they as good as for Q's 1 and 2?

In Conclusion

Locke your door at night!

That's the robust thing to do.