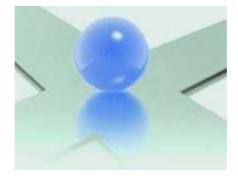
The Paradox of Choice Why MORE is LESS

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Technion

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 $^{^0\}$ lectures\talks \lib
\pdox-choice02.tex \odot Yakov Ben-Haim 18.11.2015

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1 What Makes a Good Decision?

§ Schwartz, Barry, 2004 Paradox of Choice: Why More Is Less



Figure 1: Barry Schwartz, 1946–.

§ Barry Schwartz, Yakov Ben-Haim, and Cliff Dacso, 2011, What Makes a Good Decision? Robust Satisficing as a Normative Standard of Rational Behaviour, *The Journal for the Theory of Social Behaviour*, 41(2): 209-227. Pre-print to be found on: http://info-gap.com/content.php?id=23

1.1 Choosing a College

- You've been accepted to several good universities.
- You must choose one.
- How to go about it?

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- How to go about it?
- § Multi-attribute utility analysis.

- Attributes:
 - Size.
 - \circ Location.
 - Reputation.
 - Quality of physics program.
 - Electrical Engineering Dept.
 - Social life.
 - Housing.

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- § Is this a good decision strategy?

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 - What is a good strategy?
 - What are *attributes* of a good strategy?

1.2 Severe Uncertainty: Good Strategy? Good Attributes?

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 - Choose school opportune to uncertainty.
- § Which strategy to use?

§ In order to choose a strategy we first discuss: Concepts of probability and uncertainty.

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1.3 Three Types of Probability

- § Sources:
 - Schwartz, Ben-Haim, Dacso, 2011.
 - Jonathan Baron, 2008, Thinking and Deciding.

§ What is probability?

What do the following statements mean?

• The probability of throwing "7" with 2 dice is 1/6.

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- The probability of throwing "7" with 2 dice is 1/6.
- The probability of developing prostate cancer is 0.03.
- The probability that Maccabee Tel Aviv will win championship is 1/4.

- 36 equi-probable outcomes with 2 dice.
- 6 outcomes sum to "7".
- The probability of throwing "7" with 2 dice is 1/6.
- The prob of "7" is a logical deduction.

§ Logical probability.

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- § Empirical probability.
 - Sample 10,000 healthy men, aged 45–70.
 - 300 develop prostate cancer.
 - 300/10,000 = 0.03.
 - The probability is an empirical frequency.

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- § Personal or subjective probability.
 - "Will Maccabee TA win?" ... "I think so." "How sure are you?" ... "I'd give 'em 25% chance."
 - Personal judgment.
 - Expression of confidence.

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- § Do these concepts of probability differ?

§ Empirical & Personal probability overlap. • Personal: judgment based on experience.

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- Empirical: collect relevant data.
 - Requires judgment. E.g.:
 - \circ Select "healthy" to test, health now, health history.

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- § Logical & Personal probability overlap.
 - Personal: judgment based on experience.
 - Logical:
 - \circ Deduction with rules of inference.
 - Pre-logical choice of rules of inference.
 E.g. reason by contradiction.

- § All three concepts of probability, Logical: outcome of ideal dice, Empirical: frequency of prostate cancer, Personal: Maccabee's chances,
 - Overlap but differ in meaning.

§ All three concepts of probability,

Logical: outcome of ideal dice,Empirical: frequency of prostate cancer,Personal: Maccabee's chances,

- Overlap but differ in meaning.
- Are mathematically identical: Kolmogorov axioms.

- § We discussed three types of probability.
- § Is all uncertainty probabilistic?
- § In a previous lecture¹ we claimed that ignorance is not probabilistic:
 - Monty Hall's 3-door problem.
 - Pascal's wager about God.
 - Lewis Carroll's 2-bag riddle.
 - Keynes' new material.
- § We now introduce a new distinction.

¹These examples are discussed in lecture "The Strange World of Human Decision Making", \lectures\decisions\lectures\foibles01.pdf.

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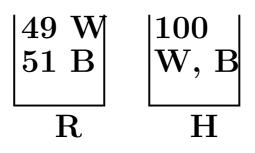
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 - Ambiguity, which we now consider.





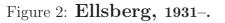
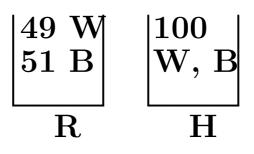


Figure 3: Ellsberg's Urns.

§ Ellsberg paradox:

• 1st experiment: win on black. Which urn?





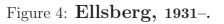


Figure 5: Ellsberg's Urns.

§ Ellsberg paradox:

- 1st experiment: win on black. Which urn?
- 2nd experiment: win on white. Which urn?



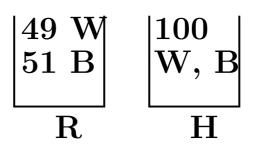




Figure 7: Ellsberg's Urns.

§ Ellsberg paradox:

- 1st experiment: win on black. Which urn?
- 2nd experiment: win on white. Which urn?
- Most folks stick with R. Sensible?



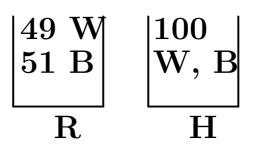




Figure 9: Ellsberg's Urns.

§ Ellsberg paradox:

- 1st experiment: win on black. Which urn?
- 2nd experiment: win on white. Which urn?
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- Types of uncertainty:
 Probability vs ambiguity.



49 W 51 B	$egin{array}{c} 100 \ W, B \end{array}$
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Figure 10: **Ellsberg**, **1931**–.

Figure 11: Ellsberg's Urns.

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- Most folks stick with R. Sensible?
- Types of uncertainty:
 - Probability vs ambiguity.
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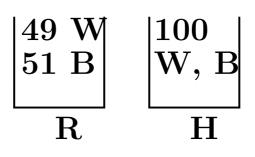


Figure 12: **Ellsberg**, **1931**–.

Figure 13: Ellsberg's Urns.

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- Types of uncertainty:
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 - Verbal information.
 - Probabilistic risk vs Knightian uncertainty.

§



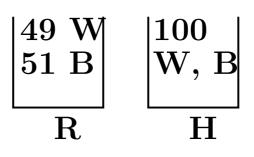


Figure 14: **Ellsberg**, **1931**–.

Figure 15: Ellsberg's Urns.

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- Most folks stick with R. Sensible?
- Types of uncertainty:
 - Probability vs ambiguity.
 - Verbal information.
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- § Now consider a different aspect: strategic interaction.





Figure 16: Cuban missile crisis, Oct 1962. U-2 rephotograph of Soviet nuclear connaissance missiles Missile transports in Cuba. and tents for fueling and maintenance are visible. Courtesy of CIA. http://en.wikipedia.org/wiki/Cuban_Missile_Crisis.

§ Strategic uncertainty.

- Outcomes depend on 2 or more agents.
- "Your" info about "Them" very limited.





Figure 17: Cuban missile crisis, Oct 1962. U-2 rephotograph of Soviet connaissance nuclear missiles Missile transports Cuba. tents for fuelin and ing and maintenance are visible. Courtesy of CIA. http://en.wikipedia.org/wiki/Cuban_Missile_Crisis.

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§ Strategic uncertainty.

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- Common knowledge: you know that they know ...
- Example: Cuban missile crisis.
 - \circ US detects nuclear missiles on Cuba.
 - How many missiles? Russian intentions unclear.
 - What should US do?

§ Ignorance is not probabilistic:²

- Monty Hall's 3-door problem.
- Pascal's wager.
- Lewis Carroll's 2-bag riddle.
- Keynes' new material.

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§ Ignorance is not probabilistic:

- Monty Hall's 3-door problem.
- Pascal's wager.
- Lewis Carroll's 2-bag riddle.
- Keynes' new material.
- § Ignorance is a gap between what we do know and what we need to know in order to make a good decision.

1.5 Responses to Severe Uncertainty

§ What decision strategy for severe uncertainty?

- Best-model optimization.
- 1-reason (lexicographic).
- Min-max (worst case).
- Robust satisficing.
- Opportune windfalling.

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- Robust satisficing.
- Opportune windfalling.
- § Paradox of choice (Barry Schwartz):
 - Under severe uncertainty, aiming for more achieves less.
 - Do you agree? Always? Sometimes?

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- § Does rob-sat differ from outcome optimization?



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 - Optimizing: Maximize caloric intake.



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 - Equity premium puzzle.
 - Home bias paradox.



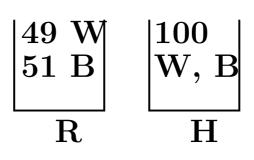


Figure 19: **Ellsberg**, **1931**–.

Figure 20: Ellsberg's Urns.

- § Humans and ambiguity: Ellsberg paradox.
 - Probabilistic risk vs uncertainty.
 - Optimizing: Maximize expected utility.



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Figure 21: **Ellsberg**, **1931**–.

Figure 22: Ellsberg's Urns.

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Figure 23: **Ellsberg**, **1931**–.

Figure 24: Ellsberg's Urns.

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 - Optimizing: Maximize expected utility.
 - Robust-satisficing: do good enough.
 - \circ Satisfice expected utility.
 - Maximize robustness to uncertainty.
 - Robust-satisficers are happier. (Schwartz)

1.6 Does Robust Satisficing Use Probability?

§ Source: Schwartz, Ben-Haim, Dacso, 2011.

Robust-satisficing implicitly uses probability:

• "Choose the more robust option."

is the same as

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- **§ Robustness question:**
 - How wrong can our estimates be and outcome of choice still acceptable?
 - The answer does not use probability. (More on this later.)

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- § Probability and Robustness are:
 - Descriptively interchangable.
 - Prescriptively distinct.
- § Robust satisficing does not use probability.

1.7 Does Robust Satisficing Use Probability? (cont.)

§ Sources:

• Yakov Ben-Haim, 2010, Uncertainty, Probability and Robust Preferences, working paper.³

• Yakov Ben-Haim, 2011, Robustness and Locke's Wingless Gentleman.⁴

³http://info-gap.com/content.php?id=23

 $^{{}^{4}}http://decisions-and-info-gaps.blogspot.com/2011/09/robustness-and-lockes-wingless.html$

§ Conflicting views on uncertainty and probability. Keynes and Carnap vs Knight and Wald.

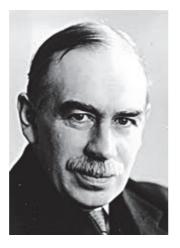


Figure 25: John Maynard Keynes, 1883–1946.

§ Probability is fundamental to uncertainty.

John Maynard Keynes asserts:⁵ "Part of our knowledge we obtain direct; and part by argument. The Theory of Probability is concerned with that part which we obtain by argument, and it treats of the different degrees in which the results so obtained are conclusive or inconclusive. ...

"The method of this treatise has been to regard subjective probability as fundamental and to treat all other relevant conceptions as derivative from this."

⁵Keynes, John Maynard, 1929, A Treatise on Probability, Macmillan and Co., London, pp.3, 281–282

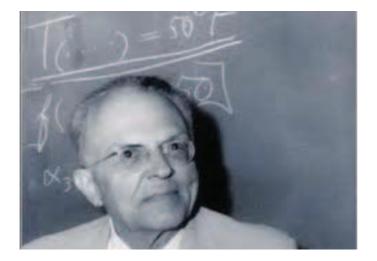


Figure 26: Rudolph Carnap, 1891–1970.

§ Probability is fundamental to uncertainty.

Among Rudolf Carnap's⁶ "basic conceptions" is the contention that "all inductive reasoning, in the wide sense of nondeductive or nondemonstrative reasoning, is reasoning in terms of probability."

⁶Carnap, Rudolf, 1962, *Logical Foundations of Probability*, 2nd ed., University of Chicago Press, p.v



Figure 27: Frank Knight, 1885–1972.

§ Probability is not fundamental to uncertainty.

Frank Knight:⁷ "Business decisions ... deal with situations which are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance. The conception of an objectively measurable probability or chance is simply inapplicable. ...

"It is this *true uncertainty* which by preventing the theoretically perfect outworking of the tendencies of competition gives the characteristic form of 'enterprise' to economic organization as a whole and accounts for the peculiar income of the entrepreneur."

⁷Knight, Frank H., 1921, *Risk, Uncertainty and Profit.* Houghton Mifflin Co. Re-issued by University of Chicago Press, 1971, pp.231–232



Figure 28: Abraham Wald, 1902–1950.

§ Probability is not fundamental to uncertainty.

Abraham Wald⁸ wrote that "in most of the applications not even the existence of ... an a priori probability distribution [on the class of distribution functions] ... can be postulated, and in those few cases where the existence of an a priori probability distribution ... may be assumed this distribution is usually unknown."

⁸Wald, A., 1945. Statistical decision functions which minimize the maximum risk, Annals of Mathematics, 46(2), 265–280, p.267

§ We consider 3 questions: (1) Does non-probabilistic robust preference between options need to assume a uniform probability distribution on underlying uncertain events?

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 - Based on set-theory representation of uncertainty: Sets, or families of sets, of events.
 - E.g. min-max (worst case) or info-gap.

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 - Based on set-theory representation of uncertainty: Sets, or families of sets, of events.
 - E.g. min-max (worst case) or info-gap.
- § Robust preference between options B and C:
 - *B* more robust than *C*.
 - Hence B "robust preferred" over C: $B \succ_{r} C$.

1.7.1 First Question

- **§ Uniform distribution nonexistent**
 - if event space unbounded.
 - Thus uniform distribution cannot underlie robust preference.

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 - if event space unbounded.
 - Thus uniform distribution cannot underlie robust preference.
 - Robust preference may be unjustified.

§ If uniform distribution exists:

• It justifies robust preference if it implies *B* more likely than *C*.

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 E.g. 3-door problem.
 - Uniform dist not necessary to justify robust pref.
 - Robust pref may still require *some* distribution.

1.7.2 Second Question

§ Does a robust preference assume *some* probability distribution on the uncertain events?

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§ Assuming a pdf could justify robust preferences. Such an assumption is not necessary.

• u is an underlying uncertain event. There is a set-model of uncertain u.

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- p(u) is a pdf on the *u*'s.

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- $p_{\rm T}(u)$ true pdf.

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- $p_{\rm T}(u)$ true pdf.

§ Our question:

Is it necessary to assume $p_{T} \in S_{B}$ to justify robust pref?

robust pref is justified probabilistically:

$$p_{\mathrm{T}} \in S_B \implies B \succ_{\mathrm{r}} C$$
 (1)

The ' \Longrightarrow ' means 'justifies' or 'warrants' or 'motivates'.

robust pref is justified probabilistically:

$$p_{\mathrm{T}} \in S_B \implies B \succ_{\mathrm{r}} C$$
 (2)

The ' \Longrightarrow ' means 'justifies' or 'warrants' or 'motivates'.

§ Converse not true:

Reasonable DM can accept \succ_r without believing $p_T \in S_B$.

robust pref is justified probabilistically:

$$p_{\mathrm{T}} \in S_B \implies B \succ_{\mathrm{r}} C$$
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§ Converse not true:

Reasonable DM can accept \succ_r without believing $p_T \in S_B$.

S Accept \succ_r if

 $p_{\mathrm{T}} \in S_B$ more likely than $p_{\mathrm{T}} \notin S_B$:

$$\operatorname{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{4}$$

robust pref is justified probabilistically:

$$p_{\mathrm{T}} \in S_B \implies B \succ_{\mathrm{r}} C$$
 (5)

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 $p_{\mathrm{T}} \in S_B$ more likely than $p_{\mathrm{T}} \notin S_B$:

$$\operatorname{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \quad \Longrightarrow \quad B \succ_{\mathrm{r}} C \tag{6}$$

§ Lower warrant of " \implies " in eq.(6) than in eq.(5), but still relevant.

§ Accept \succ_r if

$$\mathbf{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \implies B \succ_{\mathrm{r}} C \tag{7}$$

- § Converse of (7) need not hold.
- § Accept \succ_{r} if $\operatorname{Prob}(p_{T} \in S_{B}) > \frac{1}{2}$ more likely than $\operatorname{Prob}(p_{T} \in S_{B}) \leq \frac{1}{2}$: $\operatorname{Prob}\left(\operatorname{Prob}(p_{T} \in S_{B}) > \frac{1}{2}\right) > \frac{1}{2} \implies B \succ_{r} C$ (8)

§ Accept \succ_r if

$$\operatorname{Prob}(p_{\mathrm{T}} \in S_B) > \frac{1}{2} \implies B \succ_{\mathrm{r}} C$$
 (9)

§ Converse of (9) need not hold.

- § Accept $\succ_{\mathbf{r}}$ if $\operatorname{Prob}(p_{\mathbf{T}} \in S_B) > \frac{1}{2}$ more likely than $\operatorname{Prob}(p_{\mathbf{T}} \in S_B) \leq \frac{1}{2}$: $\operatorname{Prob}\left(\operatorname{Prob}(p_{\mathbf{T}} \in S_B) > \frac{1}{2}\right) > \frac{1}{2} \implies B \succ_{\mathbf{r}} C$ (10)
- § This regression can go forever. One could claim:

$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right] > \frac{1}{2}\right]\right) > \frac{1}{2}$$
$$\implies B \succ_{\mathrm{r}} C \qquad (11)$$

Converse not necessary at any step.

§ Second question was, Is:

$$p_{\rm T} \in S_B \tag{12}$$

necessary to motivate robust preferences:

$$B \succ_{\mathrm{r}} C$$
 (13)

§ Second question was, Is:

$$p_{\rm T} \in S_B \tag{14}$$

necessary to motivate robust preferences:

$$B \succ_{\mathbf{r}} C$$
 (15)

- § Summary of Q.2 answer:
 - Any of an infinity of probability beliefs would justify \succ_r to some extent:

$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right] > \frac{1}{2}\right]\right) > \frac{1}{2}$$
$$\implies B \succ_{\mathrm{r}} C \qquad (16)$$

§ Second question was, Is:

$$p_{\rm T} \in S_B \tag{17}$$

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 (18)

- § Summary of Q.2 answer:
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$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right]\right) > \frac{1}{2}\right)$$
$$\implies B \succ_{\mathrm{r}} C \qquad (19)$$

• Higher-order probability statements provide weaker justification.

$$p_{\rm T} \in S_B \tag{20}$$

necessary to motivate robust preferences:

$$B \succ_{\mathbf{r}} C$$
 (21)

- **§ Summary of Q.2 answer:**
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$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right) > \frac{1}{2}\right]\right) > \frac{1}{2}$$
$$\implies B \succ_{\mathrm{r}} C \qquad (22)$$

- Higher-order probability statements provide weaker justification.
- None of any finite sequence of prob beliefs is necessary to justify \succ_r .

$$p_{\rm T} \in S_B \tag{23}$$

necessary to motivate robust preferences:

$$B \succ_{\mathbf{r}} C$$
 (24)

- **§ Summary of Q.2 answer:**
 - Any of an infinity of probability beliefs would justify \succ_r to some extent:

$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right) > \frac{1}{2}\right]\right) > \frac{1}{2}$$
$$\implies B \succ_{\mathrm{r}} C \qquad (25)$$

- Higher-order probability statements provide weaker justification.
- None of any finite sequence of prob beliefs is necessary to justify \succ_r .
- At any step, prob belief can be deferred to next higher-order belief.

$$p_{\rm T} \in S_B \tag{26}$$

necessary to motivate robust preferences:

$$B \succ_{\mathbf{r}} C$$
 (27)

- **§ Summary of Q.2 answer:**
 - Any of an infinity of probability beliefs would justify \succ_r to some extent:

$$\operatorname{Prob}\left(\dots\left[\operatorname{Prob}\left(\operatorname{Prob}\left(p_{\mathrm{T}} \in S_{B}\right) > \frac{1}{2}\right] > \frac{1}{2}\right]\right) > \frac{1}{2}$$
$$\implies B \succ_{\mathrm{r}} C \qquad (28)$$

- Higher-order probability statements provide weaker justification.
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- At any step, prob belief can be deferred to next higher-order belief.
- § Answer to second: Nope.

$$p_{\rm T} \in S_B \tag{29}$$

necessary to motivate robust preferences:

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 (30)

- **§ Summary of Q.2 answer:**
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$$\implies B \succ_{\mathrm{r}} C \qquad (31)$$

- Higher-order probability statements provide weaker justification.
- None of any finite sequence of prob beliefs is necessary to justify \succ_r .
- At any step, prob belief can be deferred to next higher-order belief.
- § Answer to second: Nope.
- § I'm arguing like Keynes on p.107: All reasoning (not direct knowledge) is probabilistic.

1.7.3 Third Question

Does $B \succ_{r} C$ need to assume a uniform pdf on the underlying uncertain events?

§ Second question:

Does $B \succ_{\mathbf{r}} C$ assume *some* probability distribution on the uncertain events?

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§ We answered NO in both cases.

The argument was deductive, conclusive.

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§ Third question:

Is at least one probability belief, from among the infinite sequence of beliefs:

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necessary in order to justify $B \succ_{r} C$?

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necessary in order to justify $B \succ_{r} C$?

§ Answer: plausibly no.

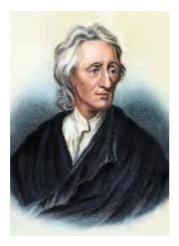


Figure 29: John Locke, 1632–1704.

§ John Locke's wingless gentleman.

"If we will disbelieve everything, because we cannot certainly know all things; we shall do muchwhat as wisely as he, who would not use his legs, but sit still and perish, because he had no wings to fly."⁹

⁹John Locke, 1706, An Essay Concerning Human Understanding, 5th edition. Roger Woolhouse, editor. Penquin Books, 1997, p.57, I.i.5



Figure 30: John Locke, 1632–1704.

§ John Locke's wingless gentleman.

"If we will disbelieve everything, because we cannot certainly know all things; we shall do muchwhat as wisely as he, who would not use his legs, but sit still and perish, because he had no wings to fly."¹⁰

§ Our situation:

• If we disbelieve all propositions in eq.(33), rejecting \succ_r is 'muchwhat as wise' as Locke's wingless gentleman.

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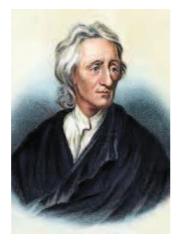


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"If we will disbelieve everything, because we cannot certainly know all things; we shall do muchwhat as wisely as he, who would not use his legs, but sit still and perish, because he had no wings to fly."¹¹

§ Our situation:

- If we disbelieve all propositions in eq.(33), rejecting \succ_r is 'muchwhat as wise' as Locke's wingless gentleman.
- Rejecting \succ_r is epistemic paralysis.

¹¹John Locke, 1706, An Essay Concerning Human Understanding, 5th edition. Roger Woolhouse, editor. Penquin Books, 1997, p.57, I.i.5

- **§ Causes of epistemic paralysis:**
 - Pdf's not known: Knightian uncertainty, info-gaps.

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- § Avoiding probabilistic epistemic paralysis:
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 - Expand conceptions of uncertainty: Adam's 20 images.

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 - Expand conceptions of uncertainty: Adam's 20 images.
- § Answer to Q.3:
 - \succ_r is epistemic last resort.
 - \succ_r is sometimes best bet. (More on this now.)

- 2 Robustness and Probability: A Short Intuitive Introduction to Proxy Theorems
- § Is robustness a good bet for "survival"?
 - Is robustness a proxy for probability?
 - Can we maximize "survival" probability without knowing probability distributions?

¹¹lectures talks lib proxy-very-shrt02.tex 19.11.2014

§ Robustness proxies for probability: Examples.



- § Foraging strategies.
 - Optimizing: Maximize caloric intake.



- § Foraging strategies.
 - Optimizing: Maximize caloric intake.
 - Robust-satisficing: survive reliably.
 - Satisfice caloric requirement.
 - Maximize robustness to uncertainty.



- § Foraging strategies.
 - Optimizing: Maximize caloric intake.
 - Robust-satisficing: survive reliably.
 - \circ Satisfice caloric requirement.
 - \circ Maximize robustness to uncertainty.
 - Robust-satisficing survives in evolution.



- § Financial market strategies.
 - Optimizing: Maximize revenue.



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 - Robust-satisficing survives in competition.
 - Equity premium puzzle.
 - Home bias paradox.



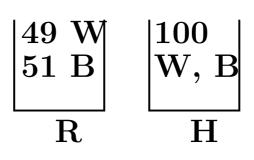


Figure 32: **Ellsberg**, **1931**–.

Figure 33: Ellsberg's Urns.

- § Humans and ambiguity: Ellsberg paradox.
 - Probabilistic risk vs uncertainty.
 - Optimizing: Maximize expected utility.



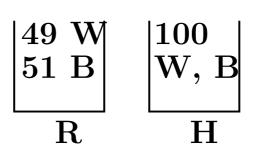


Figure 34: **Ellsberg**, **1931**–.

Figure 35: Ellsberg's Urns.

- § Humans and ambiguity: Ellsberg paradox.
 - Probabilistic risk vs uncertainty.
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 - Robust-satisficing: do good enough.
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 - \circ Maximize robustness to uncertainty.



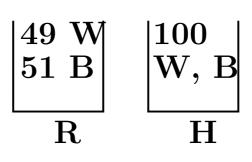


Figure 36: **Ellsberg**, **1931**–.

Figure 37: Ellsberg's Urns.

- § Humans and ambiguity: Ellsberg paradox.
 - Probabilistic risk vs uncertainty.
 - Optimizing: Maximize expected utility.
 - Robust-satisficing: do good enough.
 - \circ Satisfice expected utility.
 - Maximize robustness to uncertainty.
 - Robust-satisficers are happier.

§ Proxy theorems: Robustness proxies for Probability

§ Robust satisficing is (often) a better bet than optimizing.

3 Robust-Satisficing is a Proxy for Probability of Survival

¹¹\lectures\talks\lib\proxy02.tex 19.11.2014

§ Decision problem:

- Decision: r.
- Uncertainty: u.
- Loss function: L(r, u).
- Satisficing: $L(r, u) \leq L_c$.

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- Decision: r.
- Uncertainty: u.
- Loss function: L(r, u).
- Satisficing: $L(r, u) \leq L_c$.
- § Info-gap uncertainty model: $\mathcal{U}(h, \tilde{u}), h \ge 0$.
 - Unbounded family of nested sets.
 - Axioms:

 $\begin{array}{lll} \textbf{Contraction:} & \mathcal{U}(0,\widetilde{u}) = \{\widetilde{u}\} \\ \textbf{Nesting:} & h < h' \implies \mathcal{U}(h,\widetilde{u}) \subset \mathcal{U}(h',\widetilde{u}) \end{array}$

$$\widehat{h}(r, L_{c}) = \max\left\{h: \left(\max_{u \in \mathcal{U}(h, \widetilde{u})} L(r, u)\right) \le L_{c}\right\}$$
(34)

§

$$\widehat{h}(r, L_{c}) = \max\left\{h: \left(\max_{u \in \mathcal{U}(h, \widetilde{u})} L(r, u)\right) \le L_{c}\right\}$$
(35)

§ Robust-satisficing preferences:

$$r \succ_{\rm r} r' \quad \text{if} \quad \widehat{h}(r, L_{\rm c}) > \widehat{h}(r', L_{\rm c})$$

$$(36)$$

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$$\widehat{h}(r, L_{c}) = \max\left\{h: \left(\max_{u \in \mathcal{U}(h, \widetilde{u})} L(r, u)\right) \le L_{c}\right\}$$
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$$(38)$$

§ Probability of survival:

$$P_{\rm s}(r) = \operatorname{\mathbf{Prob}}[L(r, u) \le L_{\rm c}] = \int_{L(r, u) \le L_{\rm c}} p(u) \,\mathrm{d}u \tag{39}$$

§

$$\widehat{h}(r, L_{c}) = \max\left\{h: \left(\max_{u \in \mathcal{U}(h, \widetilde{u})} L(r, u)\right) \le L_{c}\right\}$$
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§ Probabilistic preferences:

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$$\tag{47}$$

- § Do \succ_r and \succ_p agree?
 - $\hat{h}(r, L_c)$ proxies for $P_s(r)$???
 - $\hat{h}(r, L_c) > \hat{h}(r', L_c)$ implies $P_s(r) \ge P_s(r')$???

§ Two actions, r_1 and r_2 , with robustnesses:

$$\widehat{h}(r_1, L_c) < \widehat{h}(r_2, L_c)$$

$$\widehat{h}_i = \widehat{h}(r_i, L_c), \ \mathcal{U}_i = \mathcal{U}(\widehat{h}_i, \ \tilde{q}).$$
(48)

§ U_i are nested:

Denote

$$\mathcal{U}_1 \subseteq \mathcal{U}_2 \quad \text{because} \quad \widehat{h}_1 < \widehat{h}_2$$
 (49)



\mathcal{U}_i reflect agent's beliefs.

§ Survival set:

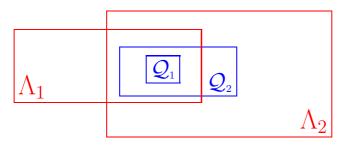
$$\Lambda(r, L_{\rm c}) = \{ u : L(r, u) \le L_{\rm c} \}$$

$$\tag{50}$$

§ Prob of survival:

$$P_{\rm s}(r) = \mathbf{Prob}[\Lambda(r, L_{\rm c})] \tag{51}$$

- § Survival sets:
 - Need not be nested.
 - Do not reflect agent's beliefs.



§ Proxy theorem need not hold.

Systems with Proxy Theorems (Each theorem with its own "fine print")

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- Decision r.
- Outcome depends on which of 2 models, A or B, is true.
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- r is fraction of budget invested in risky assets.
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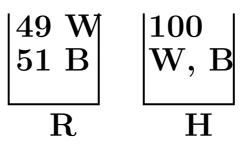


Figure 38: Ellsberg's Urns.

- § Ellsberg's paradox.
 - 2 lotteries: 1 risky, 1 ambiguous.
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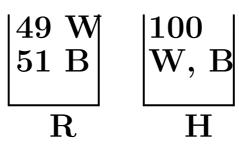


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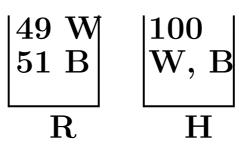


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 - 2 lotteries: 1 risky, 1 ambiguous.
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 - Choice between risk and ambiguity.
 - Ellsberg's agents prefer risky lottery both times.
 - Ellsberg's agents are robust-satisficers because:
 - Robustness is a proxy for probability, so:
 - Agents maximize probability of adequate return.

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 - $L_1(u)$ is true behavior of system.
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§ Recap:

- Many systems have proxy property.
- Many systems don't have proxy prop.
- How prevalent is the proxy property?
 - Human: economic competition.
 - Biological: foraging.
 - Physical: quantum uncertainty.

4 Robust Satisficing: Normative or Prescriptive?

§ Main Source:

Barry Schwartz, Yakov Ben-Haim, and Cliff Dacso, 2011, What Makes a Good Decision? Robust Satisficing as a Normative Standard of Rational Behaviour, *The Journal for the Theory of Social Behaviour*, 41(2): 209-227. Pre-print to be found on:

http://info-gap.com/content.php?id=23

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• Prescriptive:

Specify implementable strategies

given human and epistemic limitations.

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 - Satisficing: satisfying critical requirements.
 - People, animals, organizations don't have info and ability to optimize.
 - If they did have, they would optimize:
 - \circ Moral imperative.
 - Competitive advantage.

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prescriptive and normative: an implementable ideal.

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 - Facilitate decision making (36 jellies).

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 - Explains the paradox of choice: Why aiming at more yields less.

In Conclusion

Human decision making under uncertainty

is $In_{te}re_{sting}$

