Lecture Notes on Conservation Management

or:

Robustness, Expected Utility and the Sumatran Rhinoceros

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A Note to the Student: These lecture notes are not a substitute for the thorough study of books. These notes are no more than an aid in following the lectures.

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1 The Generic Problem: The Innovation Dilemma

¶ **Engineering design:**

- Uncertain operating conditions of device.
- Various design alternatives.
- Uncertain performance for each alternative.
- The innovative alternative seems better than the others, but is more uncertain.

What to do?

¶ **Business strategy:**

- Uncertain plans of competitive firms.
- Various development alternatives for your firm.
- Uncertain outcome for each alternative.
- The innovative alternative seems better than the others, but is more uncertain.

What to do?

¶ **Military field tactics:**

- Uncertain enemy strength and deployment.
- Various available actions.
- Uncertain outcome for each action.
- The innovative alternative seems better than the others, but is more uncertain.

What to do?

¶ **Generic problem:**

- States of the world with uncertain probabilities p_1, $p_2 \ldots$.
- Available actions a_1, $a_2 \ldots$.
- Uncertain utility v_{ij} of action i given state j.
- The innovative alternative seems better than the others, but is more uncertain.

What to do?

2 The Problem: Endangered Species

¶ The Sumatran rhinoceros is an endangered species. There are only a fairly small number of reproducing breeding pairs. We must choose a strategy which will enhance the probability of survival of the species.

¶ We will first develop the approach of **expected utility.**

¶ We will then embed the expected utility analysis in an **info-gap robust-satisficing** approach.

¶ We will deal with three basic entities:

1. **States of the world**, where p_j = probability that the world is in state j.

   The states of the world refer to the alternative possible causes of decline and disappearance of the Sumatran rhinoceros:

   (a) Poaching.

   (b) Loss of habitat.

   (c) Demographic accidents.

   (d) Disease.

2. **Actions,** denoted a_1, a_2, $\ldots$, which can be adopted to protect the rhino. These include:

   (a) Translocation of the rhino population to a new region.

   (b) Extension of the current reserve in which the rhinos live.

   (c) Captive breeding.

3. **Utilities,** v_{ij} of action a_i if the world is in state j. In our example, the utility v_{ij} will be the probability of survival of the species (for a specified duration, like a season or a decade), given that action a_i is taken when the world is in state j. Thus v_{ij} is a conditional probability. (Note that v_{ij} is not a normalized probability distribution. It may be, for instance, that the probability of survival is very low for all states of the world.)

3 Expected Utility

¶ The **expected utility** of action a_i is the average utility of that action:

$$\mathrm{E}(a_i) = \sum_j v_{ij} p_j \tag{1}$$

Since v_{ij} is the probability of survival given action a_i in state j, we see that $\mathrm{E}(a_i)$ is the probability of survival averaged over all states of the world, if action a_i is taken.

¶ The **optimal action,** $a^\star$, from the perspective of expected utility theory, is the action which maximizes the average utility:

$$a^\star = \arg\max_{a_i} \mathrm{E}(a_i) \tag{2}$$

$$= \arg\max_{a_i} \sum_j v_{ij} p_j \tag{3}$$

$a^\star$ is the action which, on average, has the highest utility (greatest probability of survival), based on the values of v_{ij} and p_j in eq.(3).

4 Uncertainties

¶ The expected utility approach is designed to deal with:

— Uncertainty in the state of the world. Hence, the terms p_j.

— Uncertainty in the survival resulting from action a_i in state j, hence the utilities v_{ij} which are conditional probabilities.

¶ However, these probabilities, p_j and v_{ij}, are themselves very imprecisely known. There are large **info-gaps** between the best estimates and the true values of these quantities.

¶ Consider the idea of **fractional error** of the estimate, $\widetilde{p}_j$:

- p_j = **unknown** true value of probability.
- $\widetilde{p}_j$ = **known** estimated value of probability.

  Fractional error of the estimate:

$$\left| \frac{p_j - \widetilde{p}_j}{\widetilde{p}_j} \right| \tag{4}$$

- We make two assertions based on our knowlege and our ignorance:
  - The fractional error is bounded: $\left| \frac{p_j - \widetilde{p}_j}{\widetilde{p}_j} \right| \leq h$.
  - The value of the bound, h, is unknown: $h \geq 0$.
- We may make other assertions, such as 'p_j is a probability so $0 \leq p_j \leq 1$'.
- Such assertions lead to the following fractional error model for uncertainty in the p_j's:

$$\mathcal{P}(h, \widetilde{p}) = \left\{ p : \ \sum_j p_j = 1, \ 0 \leq p_j \leq 1, \ \left| \frac{p_j - \widetilde{p}_j}{\widetilde{p}_j} \right| \leq h \right\}, \quad h \geq 0 \tag{5}$$

¶ We will represent the uncertainties in p_j and v_{ij} by the following fractional-error info-gap models $\mathcal{P}(h, \widetilde{p})$ and $\mathcal{V}(h, \widetilde{v})$. Eq.(6) is equivalent to eq.(5).

$$\mathcal{P}(h, \widetilde{p}) = \left\{ p : \ \sum_j p_j = 1. \ \max[0, \ (1-h)\widetilde{p}_j] \ \leq \ p_j \ \leq \ \min[1, \ (1+h)\widetilde{p}_j], \ j = 1, \ 2, \ \ldots \right\}, \quad h \geq 0 \tag{6}$$

$$\mathcal{V}(h, \widetilde{v}) = \{ v : \ \max[0, \ (1-h)\widetilde{v}_{ij}] \ \leq \ v_{ij} \ \leq \ \min[1, \ (1+h)\widetilde{v}_{ij}], \ i = 1, \ 2, \ldots j = 1, \ 2, \ldots \}, \quad h \geq 0 \tag{7}$$

These are fractional-error info-gap models, adapted to the specific case of probabilities. In particular:

1. Since p_j and v_{ij} are probabilities, they must lie in the interval $[0, \ 1]$.

2. The probability distribution p_j is normalized on j.

3. The probability distribution v_{ij} is not normalized on j:

   It could be that $\sum_j v_{ij} \le 1$, which occurs if all the survival probabilities are very small.

   It could be that $\sum_j v_{ij} \ge 1$, which occurs if all the survival probabilities are very large.

5 Robustness

¶ Given estimates $\widetilde{p}$ and $\widetilde{v}$ of the probabilities, we can estimate the expected utility of any action a_i, $\mathrm{E}(a_i, \widetilde{p}, \widetilde{v})$.

¶ For any other choice of the probabilities, p and v, the expected utility is $\mathrm{E}(a_i, p, v)$.

¶ Since these estimates, $\widetilde{p}$ and $\widetilde{v}$, are very uncertain, we do not have confidence that the actual utility which is expected to result from action a_i equals $\mathrm{E}(a_i, \widetilde{p}, \widetilde{v})$.

That is, we have every reason to believe that, for many choices of p and v, and especially for the true choice:

$$\mathrm{E}(a_i, p, v) \neq \mathrm{E}(a_i, \widetilde{p}, \widetilde{v}) \tag{8}$$

¶ Let E_{c} be the lowest level of expected utility (least average probability of survival of the species) which we are willing to accept. This is the idea of **satisficing.**

¶ The robustness of action a_i, to uncertainties in the probabilities p_j and v_{ij}, is the greatest horizon of uncertainty h up to which adequate expected utility, E_{c}, is obtained for any realization of the probabilities:

$$\widehat{h}(a_i, E_{\mathrm{c}}) = \max \left\{ h : \left(\min_{\substack{p \in \mathcal{P}(h, \widetilde{p}) \\ v \in \mathcal{V}(h, \widetilde{v})}} \sum_j v_{ij} p_j \right) \geq E_{\mathrm{c}} \right\} \tag{9}$$

¶ The robust optimal action, $\widehat{a}(E_{\mathrm{c}})$, maximizes the robustness and satisfices the expected utility at the value E_{c}:

$$\widehat{a}(E_{\mathrm{c}}) = \arg\max_{a_i} \widehat{h}(a_i, E_{\mathrm{c}}) \tag{10}$$

¶ The robust-optimal action, $\widehat{a}(E_{\mathrm{c}})$, depends on the aspiration for survival, E_{c}.

¶ The robust-optimal action, $\widehat{a}(E_{\mathrm{c}})$, is very likely to be different from $a^\star$, the action which maximizes the best-estimate of the expected utility.

¶ The robustness, $\widehat{h}(a_i, E_{\mathrm{c}})$ in eq.(9), combines the following 3 basic components:

- **System model:** $E(a_i, p, v)$.
- **Performance requirement:** $E(a_i, p, v) \geq E_{\mathrm{c}}$.
- **Uncertainty models:** $\mathcal{P}(h, \widetilde{p})$, $\mathcal{V}(h, \widetilde{v})$.

6 Example

State (Cause of decline)	Probability p_j of that state	Cond. Prob. v_{1j} (Translocation) (a_1)	Cond. Prob. v_{2j} (New reserve) (a_2)	Cond. Prob. v_{3j} (Captive breeding) (a_3)
Poaching	0.1	0.3	0.25	0.9
Loss of habitat	0.3	0.1	0.2	0.2
Demographic accidents	0.5	0.05	0.09	0.01
Disease	0.1	0.1	0.1	0.4
Expected utility		$\sum_j v_{1j}p_j = 0.095$	$\sum_j v_{2j}p_j = 0.14$	$\sum_j v_{3j}p_j = 0.195$

Table 1: Estimated probabilities.

¶ From table 1 we see that action a_3, captive breeding, has the greatest expected utility. The EU approach therefore recommends action a_3. This is in fact the conservation strategy which has been recommended by conservation biologists who have studied the sumatran rhinoceros problem.

¶ However, we know that the robustness of maximal expectations is zero! That is:

$$0 = \widehat{h}(a_1, 0.095) = \widehat{h}(a_2, 0.14) = \widehat{h}(a_3, 0.195) \tag{11}$$

That is, action a_1 cannot be relied upon to result, on the average, in utility 0.095.
Likewise, action a_2 cannot be relied upon to result, on the average, in utility 0.14.
Likewise, action a_3 cannot be relied upon to result, on the average, in utility 0.195.
Infinitesimal errors in p or v may result in either better, or worse, average probability of survival.

¶ How much performance (probability of survival) must be foregone in order to obtain a reliable probability of survival?

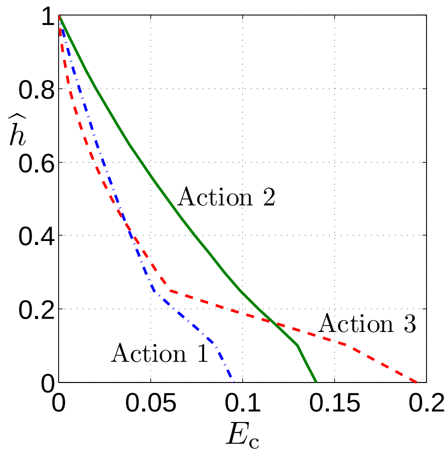

Figure 1: Robustness curves for actions 1, 2 and 3.

¶ From the figure we note:

1. The trade-off between performance (demanded expected utility E_c) and robustness to uncertainty.

2. The nominal optimal action, according to expected-utility theory, $a^\star = a_3$ (captive breeding), has zero robustness.

3. In fact the robustnesses of all the nominal expected utilities are zero.

4. Reversal of preference:

   (a) For $E_c > 0.12$, the most robust action is captive breeding (a_3).

   (b) For $E_c < 0.12$, the most robust action is extension of the current reserve (a_2).

   (c) For $E_c < 0.04$ there is a preference reversal between a_1 and a_3, but a_2 is still the option of choice.

   In other words, the robust-optimal choice of an action depends on the performance which is required.