18. **General inflation.** (p.72) (Based on DeGarmo, 9-2, p.395) If the average general inflation is expected to be 8% per year, how many years will it take to reduce the currency's purchasing power by one-half? Generalizing this, how long would it take to reduce the currency's purchasing power by a factor of $1/n$?

19. **Comparing alternatives under inflation.** (p.72) (DeGarmo, 9-4, p.396) The annual expenses for two programs are evaluated on different bases, as shown in table 1.

End of Year	Alternative A Nominal $	Alternative B Real $
1	−120,000	−100,000
2	−132,000	−110,000
3	−148,000	−120,000
4	−160,000	−130,000

Table 1: Problem 19.

If the average general price inflation is expected to be 6% per year, and the real rate of interest is 9% per year, evaluate the PW of the two alternatives. Which has the least negative PW?

Solution to Problem 18, General inflation (p.15).

¶ The nominal sum A_0 buys a basket of goods, B, in year zero. The annual price inflation of these goods is f. Thus the nominal sum required to purchase B in year k is:

$$A_k = (1+f)^k A_0 \tag{210}$$

The purchasing power of the currency is reduced by $\frac{1}{2}$ if twice as many dollars are need in year k, so:

$$A_k = 2A_0 \tag{211}$$

Thus:

$$\frac{1}{2} = \frac{A_0}{A_k} = (1+f)^{-k} \implies \ln\frac{1}{2} = -k\ln(1+f) \implies k = \frac{\ln 2}{\ln(1+f)} = 9.006 \sim 9 \tag{212}$$

¶ Here is another way of looking at the problem (and getting the same answer). Let A_k be the nominal value (number of dollar bills) needed in year k to buy the basket of goods B. The real value in year k is:

$$R_k = (1+f)^{-k} A_k \tag{213}$$

The real value of B is constant:

$$R_k = R_0 \tag{214}$$

Real and nominal values are equal in the base year:

$$R_0 = A_0 \tag{215}$$

So eq.(213) is:

$$A_0 = (1+f)^{-k} A_k \tag{216}$$

We seek the year, k, in which the purchasing power of \$1 is reduced by half, so that twice as many dollars are needed to purchase B:

$$A_k = 2A_0 \tag{217}$$

Thus eq.(216) becomes:

$$\frac{1}{2} = (1+f)^{-k} \tag{218}$$

We now continue as in eq.(212).

¶ Find k for which $A_0/A_k = 1/n$ (see fig. 7):

$$\frac{1}{n} = (1+f)^{-k} \implies \ln\frac{1}{n} = -k\ln(1+f) \implies k = \frac{\ln n}{\ln(1+f)} \tag{219}$$

Solution to Problem 19, Comparing alternatives under inflation (p.15).

First consider alternative A. The real and nominal value of the expense in year k are related as:

$$R_k^{\mathrm{A}} = (1+f)^{-k} A_k^{\mathrm{A}} \tag{220}$$

The PW of alternative A is:

$$\begin{aligned} \mathrm{PW}_{\mathrm{A}} &= \sum_{k=1}^{N} (1+i_{\mathrm{r}})^{-k} R_k^{\mathrm{A}} && (221) \\ &= \sum_{k=1}^{N} (1+i_{\mathrm{r}})^{-k} (1+f)^{-k} A_k^{\mathrm{A}} && (222) \\ &= -388,476.01 && (223) \end{aligned}$$

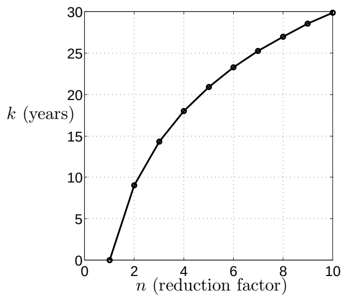

Figure 7: k (years) vs n (reduction factor), problem 18, eq.(219).

Now consider alternative B. The PW of alternative B is:

$$\text{PW}_\text{B} = \sum_{k=1}^{N} (1+i_\text{r})^{-k} R_k^\text{B} \tag{224}$$

$$= -369,085.21 \tag{225}$$

Alternative B has lower PW of its expenses and thus, from this point of view, is preferred.