20. **Salary erosion from inflation.** (p.74) (Based on DeGarmo, 9-6, p.396) An engineer received the nominal salaries shown in table 2 over the past 4 years, with inflation, f_k, in % indicated for each year.

    (a) If f_k is a measure of the general price inflation, evaluate the annual salaries in real year-0 dollars.

    (b) Now suppose that the inflation values in table 2 are estimates, where each estimate could err by $\pm$10% or more. You require that the real income in each year, $k = 1, \ldots, 4$, not be less than a specified value $R_{k,\mathrm{c}}$. Derive an expression for the inverse of the robustness function for each year.

End of Year k	Nominal salary A_k ($)	f_k
1	34,000	7.1%
2	36,200	5.4%
3	38,800	8.9%
4	41,500	11.2%

Table 2: Data for problem 20.

Solution to Problem 20, Salary erosion from inflation (p.16).

(20a) The year 0 real salaries are calculated as follows. See results in table 10 on p.74.

• **Nominal income from end of year 1:** The year 0 nominal equivalent of the nominal income in year 1, correcting for inflation in year 1, is:

$$A_{0,1} = (1+f_1)^{-1} A_1 \tag{226}$$

Nominal and real income in year-0 are the same, so the **real year 0 income from year 1 is:**

$$R_{0,1} = A_{0,1} = (1+f_1)^{-1} A_1 \tag{227}$$

• **Nominal income from end of year 2:** The year 1 nominal equivalent of the nominal income in year 2, correcting for inflation in year 2, is:

$$A_{1,2} = (1+f_2)^{-1} A_2 \tag{228}$$

The year 0 nominal equivalent of nominal income $A_{1,2}$, correcting for inflation in year 1, is:

$$A_{0,2} = (1+f_1)^{-1} A_{1,2} = (1+f_1)^{-1}(1+f_2)^{-1} A_2 \tag{229}$$

Nominal and real income in year 0 are the same, so the **real year 0 income from year 2 is:**

$$R_{0,2} = A_{0,2} = (1+f_1)^{-1}(1+f_2)^{-1} A_2 \tag{230}$$

• **Nominal income from end of year k:** Generalizing eq.(229), the nominal income in year 0 from the income in year k is:

$$A_{0,k} = A_k \prod_{j=1}^{k} (1+f_j)^{-1} \tag{231}$$

Nominal and real income in year-0 are the same, so the **real year 0 income from year k is:**

$$R_{0,k} = A_{0,k} = A_k \prod_{j=1}^{k} (1+f_j)^{-1} \tag{232}$$

The nominal and real salaries are shown in table 10.

Year, k	$\prod_{j=1}^{k}(1+f_j)^{-1}$	A_k	R_k
1	0.9337	34,000	31,746
2	0.8859	36,200	32,068
3	0.8135	38,800	31,563
4	0.7315	41,500	30,359

Table 10: Solution to problem 20a.

(20b) An info-gap model for uncertain inflation is:

$$\mathcal{U}(h) = \left\{ f : \ f_k > -1, \ \left| \frac{f_k - \widetilde{f}_k}{s_k} \right| \le h, \ k = 1, \ldots, 4 \right\}, \quad h \ge 0 \tag{233}$$

where $\widetilde{f}_k$ is the estimated inflation in year k and $s_k = \varepsilon \widetilde{f}_k$.

The performance requirement is:

$$R_{0,k} \ge R_{kc} \tag{234}$$

The robustness for year k is defined as:

$$\widehat{h}_k = \max \left\{ h : \left(\min_{f \in \mathcal{U}(h)} R_{0,k}(f) \right) \geq R_{k\mathrm{c}} \right\} \tag{235}$$

The inner minimum, $m_k(h)$, is the inverse of the robustness and occurs when each f_k is as large as possible at horizon of uncertainty h: $f_k = \widetilde{f}_k + s_k h = \widetilde{f}_k + \varepsilon \widetilde{f}_k h = (1 + \varepsilon h)\widetilde{f}_k$. Thus, from eq.(232):

$$m_k = A_k \prod_{j=1}^{k} \left[1 + (1 + \varepsilon h) \widetilde{f}_j \right]^{-1} \tag{236}$$