17 Behavioral Response to Feedback

17.1 Introduction

§ The Israel Electric Corporation (IEC) has adopted the practice of **reporting to consumers their level of energy consumption compared to a local mean.** The IEC's goal, of course, is to encourage energy conservation, but the outcome may be different in the long run. Consider the following:

1. The **Lo family** gets feedback indicating that their energy consumption is **below the average**, and the **Hi family's** feedback shows their consumption is **above the average.**
2. One might expect that the **Lo family will tend to increase** their consumption since they are already relatively conservative. Likewise, one might expect a tendency of the **Hi family to reduce** consumption.
3. In the spirit of **Kahneman-Tversky**, let's invoke an **asymmetry between positive and negative reward** as in fig. 44. The Lo family gets positive reward by increasing consumption by the amount U, while the Hi family gets negative reward by decreasing consumption by the amount D. The Kahneman-Tversky asymmetry would suggest that U will tend to be greater than D.

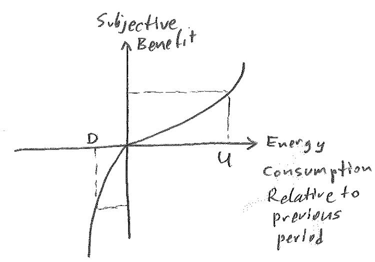

Figure 44: Kahneman-Tversky's asymmetric subjective utility function.

4. Consequently, **the average consumption will tend to drift upward over time.** In other words, the IEC feedback may have the opposite effect from what was intended.
5. The behavior of the Lo and Hi families demonstrates a **"reversion to the mean"**, as one might expect. However, the Kahneman-Tversky asymmetry implies that this **reversion is asymmetric** and may cause a long-range upward drift of the mean.
6. This is somewhat similar to the **Lucas critique:** populations tend to act, inadvertently and without coordination, to contravene long-range policy goals.
7. This "story" must be treated with caution. Life, and people, are more complicated. Nonetheless, treated as an hypothesis, it might be worth exploring, because if it is true then the IEC's feedback policy is misguided (or maybe intentional? Noooo. :)
8. The **asymmetry can, however, be manipulated** by changing the reference point with respect to which high and low consumption are defined. Suppose that comparison with the mean or the median causes long-term upward drift of the mean. In this case, comparison with a lower value, say the 30th percentile, could cause long-term drift downward because now fewer people feel they are conserving. Of course, predicting what reference point will cause stability, or drift up or down at a particular rate, is highly uncertain. One can then, of course, do an **info-gap robustness analysis to manage this uncertainty.**

9. This problem can be generalized from the specific case of energy conservation. One can think of savings vs. consumption, or risky vs. risk-free investment, or consumption of domestic vs. foreign products, etc. In some cases one may want to decrease consumption (e.g. of energy), and in others one may want to increase consumption (e.g. of domestic products).

17.2 Further Examples of Behavioral Response to Feedback

§ **Profiling.** The economic theory of crime views criminals as rational decision makers, implying elastic response to law enforcement. That is, more enforcement implies less crime. Different groups have different elasticities of response to enforcement. This suggests that group-dependent elasticities can be exploited for efficient allocation of enforcement resources: profiling. However, profiling can augment both number of arrests and total crime because non-profiled groups will increase their criminality. Elasticities are highly uncertain, so prediction is difficult and uncertainty must be accounted for in designing a profiling strategy.[11]

§ **Marginal tax revenue.** Governments fund their activities by taxing the public. Governments can increase their total budget by increasing the marginal income tax rate. However, greater marginal income tax rate decreases the incentive to work, especially at the margin (that extra hour, or that extra job, become less attractive). Thus increasing the marginal income tax rate causes a decrease in total earning by the public, and can cause a net decrease in tax revenue.

§ **Lucas critique.** Keynesian economic models are, traditionally, used to formulate macroeconomic policy based on historical data about supply and demand curves and other aggregate economic data. Robert Lucas pointed out that behavior by consumers and firms can change in response to changes in policy. Hence traditional Keynesian policy analysis, based on aggregated historical data, is unreliable. Lucas suggested that one must incorporate microeconomic dimensions to the model in order to account for this response to policy. One might be able to avoid the microeconomic dimension by treating the macro models as uncertain, and robustifying against this uncertainty.

§ **Principal-agent contract bidding.** An employer (the 'principal') offers a contract to a prospective employee (the 'agent'). If the employee accepts the contract, the employee's effort will bring benefit to the employer. However, the extent of the employee's effort depends on the employee's response to the incentives provided in the contract. The employer is uncertain about the employee's response to these incentives. That is, the employer is uncertain about the employee's response to the future feedback provided in the contract.[12]

§ **Arms race and the security dilemma.** Consider two countries that fear each other's military capabilities. If one country extends its military capability, the other country may view this as purely defensive and take no action. Or, the other country may view this as offensive build up and extend its military capability in response. The security dilemma is the potential for a spiral enlargement of military capability by both countries that can lead to reduced security for both, or even lead to armed conflict.

[11] Lior Davidovitch and Yakov Ben-Haim, 2011, Is your profiling strategy robust? *Law, Probability and Risk*, 10: 59–76.

[12] Yakov Ben-Haim, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty*, 2nd edition, Academic Press, London, section 9.3.

17.3 Formulation

We now return to the IEC example.

§ **Definitions:**

ρ = a reference consumption (of energy) in time interval 1. This value is revealed to the consumers at the end of the time interval. This is the feedback to which consumers respond.

c_1 = the consumption of energy (kW hr) in time interval 1, which varies from consumer to consumer.

$n(c_1)\,\mathrm{d}(c_1)$ = number of consumers whose consumption in time interval 1 was in the interval $[c_1,\ c_1 + \mathrm{d}c_1]$. Thus $n(c_1)$ is a number density, 1/(kW hr). This function is known from historical data. Or, it is known at the end of time interval 1 because the consumptions of all consumers are observed.

Γ_1 = the total consumption in time interval 1, which equals:

$$\Gamma_1 = \int_0^\infty c_1 n(c_1)\,\mathrm{d}c_1 \tag{316}$$

$f(c_1, \rho)$ = consumption in the next time interval of a consumer whose prior consumption was c_1. This function depends on ρ because the consumer's behavior responds to this feedback. $f(c_1, \rho)$ is non-negative but uncertain.

$\widetilde{f}(c_1, \rho)$ = the putative estimated consumer response function, which is known and non-negative.

$\mathcal{U}(h)$ = an info-gap model for uncertainty in the function $f(c_1, \rho)$.

Γ_2 = the total consumption in time interval 2, which equals:

$$\Gamma_2 = \int_0^\infty f(c_1, \rho) n(c_1)\,\mathrm{d}c_1 \tag{317}$$

§ **Asymmetry.** $f(c_1, \rho)$ might have the asymmetry properties referred to in item 3 and fig. 44, p.93. Specifically, it might be that the increase in consumption by conservative consumers exceeds the decrease in consumption by excessive consumers. For any positive change in consumption, δ, define:

$\rho + \delta$ = excessive consumption in the 1st period.

$f(\rho + \delta, \rho)$ = that consumer's reduced consumption in the 2nd period: $f(\rho + \delta, \rho) < \rho + \delta$.

$\rho - \delta$ = under-consumption in the 1st period.

$f(\rho - \delta, \rho)$ = that consumer's enhanced consumption in the 2nd period: $f(\rho - \delta, \rho) > \rho - \delta$.

That is, defining U and D as in item 3 and fig. 44, p.93, for any positive increment of consumption, δ:

$$\underbrace{\rho + \delta - f(\rho + \delta, \rho)}_{D>0} < \underbrace{f(\rho - \delta, \rho) - (\rho - \delta)}_{U>0} \tag{318}$$

This implies:

$$\frac{f(\rho + \delta, \rho) + f(\rho - \delta, \rho)}{2} > \rho \tag{319}$$

Thus $f(c,\rho)$ vs. c is upward-concave.

We might expect that, when $\delta = 0$, the consumption does not change as a result of the feedback:

$$f(\rho,\rho) = \rho \tag{320}$$

§ **Performance requirement.** In general, there are two possibilities: we want total consumption to either decrease or increase by a non-negative quantity ε.
The total consumption must **decrease** by at least ε:

$$\Gamma_1 - \Gamma_2 \geq \varepsilon \tag{321}$$

The total consumption must **increase** by at least ε:

$$\Gamma_2 - \Gamma_1 \geq \varepsilon \tag{322}$$

§ **Definition of the robustness for decreasing consumption** by at least ε, from eq.(321):

$$\widehat{h}(\varepsilon,\rho) = \max\left\{h: \ \left(\min_{f\in\mathcal{U}(h)} [\Gamma_1 - \Gamma_2]\right) \geq \varepsilon\right\} \tag{323}$$

§ **Definition of the robustness for increasing consumption** by at least ε, from eq.(322):

$$\widehat{h}(\varepsilon,\rho) = \max\left\{h: \ \left(\min_{f\in\mathcal{U}(h)} [\Gamma_2 - \Gamma_1]\right) \geq \varepsilon\right\} \tag{324}$$

17.4 Robustness for Decreasing Consumption; Fractional Error Info-Gap Model I

§ The info-gap model for uncertainty in the consumers' responses is:

$$\mathcal{U}(h) = \left\{f(c_1,\rho): \ f(c_1,\rho) \geq 0, \ \left|\frac{f(c_1,\rho) - \widetilde{f}(c_1,\rho)}{\widetilde{f}(c_1,\rho)}\right| \leq h\right\}, \quad h \geq 0 \tag{325}$$

Note that we do not require the consumption functions to obey the conditions in eqs.(319) and (320).

§ Let $m(h)$ denote the inner minimum in the definition of the robustness, eq.(323). Note that:

$$\Gamma_1 - \Gamma_2 = \int_0^\infty [c_1 - f(c_1,\rho)]\, n(c_1)\, \mathrm{d}c_1 \tag{326}$$

§ From eq.(326) we see that $m(h)$ occurs when $f(c_1,\rho)$ is as large as possible at horizon of uncertainty h, namely:

$$f(c_1,\rho) = (1+h)\widetilde{f}(c_1,\rho) \tag{327}$$

§ We now find the inner minimum in the robustness to be:

$$\begin{aligned} m(h) &= \int_0^\infty \left[c_1 - (1+h)\widetilde{f}(c_1,\rho)\right] n(c_1)\, \mathrm{d}c_1 && (328) \\ &= \Gamma_1 - (1+h)\widetilde{\Gamma}_2(\rho) && (329) \end{aligned}$$

where $\widetilde{\Gamma}_2(\rho)$ is the putative value of the total consumption in the 2nd time interval, and it depends on the reference consumption, ρ.

§ The performance requirement is $m(h) \geq \varepsilon$, where $\varepsilon > 0$, namely:

$$\Gamma_1 - (1+h)\widetilde{\Gamma}_2 \geq \varepsilon \tag{330}$$

§ Solving for h in eq.(330) at equality yields the robustness for decreasing consumption:

$$\frac{\Gamma_1 - \varepsilon}{\widetilde{\Gamma}_2} = (1+h) \quad \Longrightarrow \quad \boxed{\widehat{h}(\varepsilon, \rho) = \begin{cases} \frac{\Gamma_1 - \varepsilon}{\widetilde{\Gamma}_2(\rho)} - 1 & \text{if } \varepsilon \leq \Gamma_1 - \widetilde{\Gamma}_2(\rho) \\ 0 & \text{else} \end{cases}} \tag{331}$$

§ ε is the required positive decrement in total consumption. Thus, if the putative 2nd-period total consumption, $\widetilde{\Gamma}_2(\rho)$, exceeds the 1st period total consumption, Γ_1, then the robustness in eq.(331) is zero.

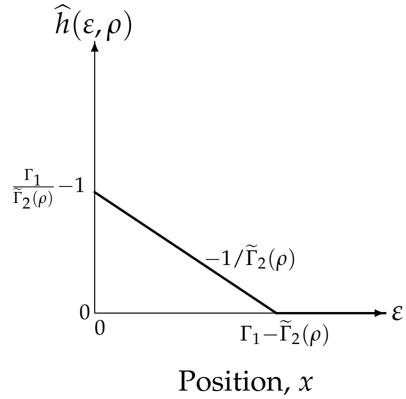

Figure 45: Robustness curve for decreasing the consumption, eq.(331), showing zeroing and trade off.

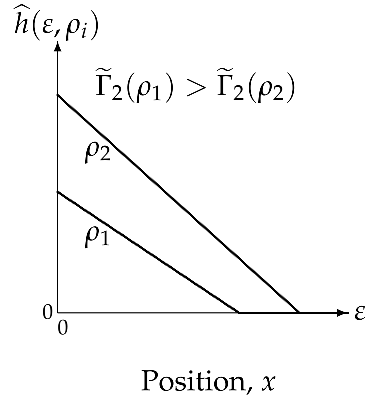

Figure 46: Two robustness curves for decreasing the consumption, with different values of the reference consumption.

§ The robustness function in eq.(331) is shown schematically in fig. 45, p.97, demonstrating the properties of trade off and zeroing.

§ Fig. 46, p.97, shows robustness curves for two different values of the reference consumption. Reference value ρ_2 is putatively better than reference value ρ_1 because ρ_2 results in a greater putative reduction in consumption (horizontal intercept):

$$\Gamma_1 - \widetilde{\Gamma}_2(\rho_2) > \Gamma_1 - \widetilde{\Gamma}_2(\rho_1) \tag{332}$$

However, the putative consumptions have zero robustness and therefore are not a good basis for comparing these alternatives.

§ Nonetheless, fig. 46 shows that reference value ρ_2 is more robust than ρ_1 for all values at which ρ_2 has postive robustness. ρ_2 is **robust dominant** over ρ_1. Thus ρ_2 is preferred over ρ_1 based on robustness. Whether ρ_2 is actually acceptable depends on judgment of whether its robustness is great enough at an acceptable reduction of consumption.

§ Summarizing fig. 46, we see that a change in the reference consumption, ρ, that causes a **decrease** in total putative consumption, $\widetilde{\Gamma}_2(\rho_2) < \widetilde{\Gamma}_2(\rho_1)$, also causes a **decrease** in the **cost of robustness:** the robustness curve for ρ_2 is steeper than for ρ_1.

§ The previous observation implies a **re-enforcing impact on the robustness** of the two aspects. Lower $\widetilde{\Gamma}_2(\rho_2)$ shifts the robustness curve to the right, and lower cost of robustness makes the ρ_2 robustness curve steeper. Hence, the robustness curves do not cross one another, as we see in fig. 46.

17.5 Robustness for Decreasing Consumption; Fractional Error Info-Gap Model II

§ The info-gap model for uncertainty in the consumers' responses is modified from eq.(325), p.96, as follows:

$$\mathcal{U}(h) = \left\{ f(c_1,\rho) : \ f(c_1,\rho) \geq 0, \ \left| \frac{f(c_1,\rho) - \widetilde{f}(c_1,\rho)}{w\widetilde{f}(c_1,\rho)} \right| \leq h \right\}, \quad h \geq 0 \tag{333}$$

where w is a positive error weight assessing a degree of uncertainty. As before, we do not require the consumption functions to obey the conditions in eqs.(319) and (320).

§ Let $m(h)$ denote the inner minimum in the definition of the robustness, eq.(323). As in eq.(326):

$$\Gamma_1 - \Gamma_2 = \int_0^\infty [c_1 - f(c_1,\rho)]\, n(c_1)\, \mathrm{d}c_1 \tag{334}$$

§ From eq.(334) we see that $m(h)$ occurs when $f(c_1,\rho)$ is as large as possible at horizon of uncertainty h, namely:

$$f(c_1,\rho) = (1+wh)\widetilde{f}(c_1,\rho) \tag{335}$$

§ We now find the inner minimum in the robustness to be:

$$\begin{aligned} m(h) &= \int_0^\infty \left[c_1 - (1+wh)\widetilde{f}(c_1,\rho) \right] n(c_1)\, \mathrm{d}c_1 && (336) \\ &= \Gamma_1 - (1+wh)\widetilde{\Gamma}_2(\rho) && (337) \end{aligned}$$

where $\widetilde{\Gamma}_2(\rho)$ is the putative value of the total consumption in the 2nd time interval, and it depends on the reference consumption, ρ.

§ The performance requirement is $m(h) \geq \varepsilon$, where $\varepsilon > 0$, namely:

$$\Gamma_1 - (1+wh)\widetilde{\Gamma}_2 \geq \varepsilon \tag{338}$$

§ Solving for h in eq.(338) at equality yields the robustness:

$$\frac{\Gamma_1 - \varepsilon}{\widetilde{\Gamma}_2} = (1+wh) \quad \Longrightarrow \quad \boxed{\widehat{h}(\varepsilon,\rho) = \begin{cases} \frac{1}{w}\left(\frac{\Gamma_1 - \varepsilon}{\widetilde{\Gamma}_2(\rho)} - 1 \right) & \text{if } \varepsilon \leq \Gamma_1 - \widetilde{\Gamma}_2(\rho) \\ 0 & \text{else} \end{cases}} \tag{339}$$

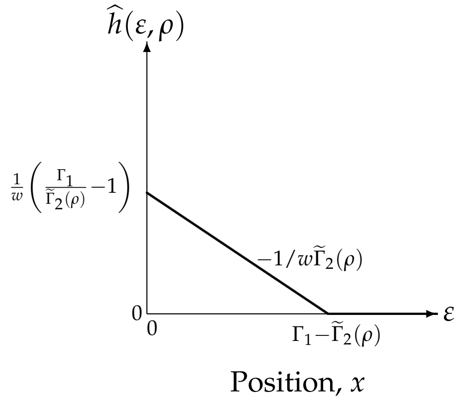

Figure 47: Robustness curve for decreasing the consumption, eq.(339), showing zeroing and trade off.

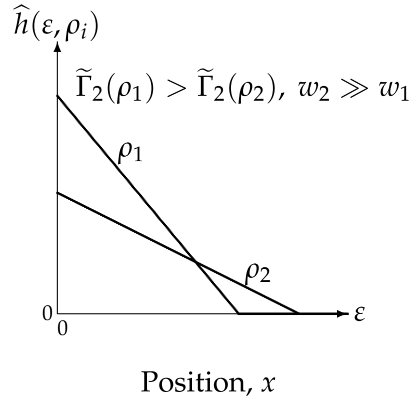

Figure 48: Two robustness curves for decreasing the consumption, with different values of the reference consumption and different uncertainty weights.

§ ε is the required positive decrement in total consumption. Thus, if the putative 2nd-period total consumption, $\widetilde{\Gamma}_2(\rho)$, exceeds the 1st period total consumption, Γ_1, then the robustness in eq.(339) is zero.

§ The robustness function in eq.(339) is shown schematically in fig. 47, p.99, demonstrating the properties of trade off and zeroing.

§ Fig. 48, p.99, shows robustness curves for two different values of the reference consumption, ρ_i, and uncertainty weights w_i. Reference value ρ_2 **is putatively better than reference value** ρ_1 because ρ_2 results in a greater putative reduction in consumption (horizontal intercept):

$$\Gamma_1 - \widetilde{\Gamma}_2(\rho_2) > \Gamma_1 - \widetilde{\Gamma}_2(\rho_1) \tag{340}$$

However, reference value ρ_1 **is less uncertain than reference value** ρ_2:

$$w_1 \ll w_2 \tag{341}$$

§ We face a **dilemma:** option 1 is putatively worse but less uncertain than option 2.

§ Eqs.(340) and (341) cause the robustness curves cross one another; fig. 48. Option 2 may be *new and innovative:* putatively better but more uncertain. The entails the possibility for **preference reversal** between the options.

§ Summarizing fig. 48, we see that a change in the reference consumption, ρ and uncertainty weight w, can cause a **decrease** in total putative consumption, $\widetilde{\Gamma}_2(\rho_2) < \widetilde{\Gamma}_2(\rho_1)$, but also cause an **increase** in the cost of robustness: the robustness curve for ρ_2 is less steep than for ρ_1.

17.6 Robustness for Increasing Consumption; Fractional Error Info-Gap Model

§ The info-gap model for uncertainty in the consumers' responses is eq.(325), as in section 17.4.

§ Let $m(h)$ denote the inner minimum in the definition of the robustness, eq.(324), p.96. Note that:

$$\Gamma_2 - \Gamma_1 = \int_0^\infty [f(c_1,\rho) - c_1]\, n(c_1)\, dc_1 \tag{342}$$

§ From eq.(342) we see that $m(h)$ occurs when $f(c_1,\rho)$ is as small as possible at horizon of uncertainty h, namely:

$$f(c_1,\rho) = (1-h)^+ \widetilde{f}(c_1,\rho) \tag{343}$$

where $x^+ = x$ if $x > 0$ and equals 0 otherwise.

§ We now find the inner minimum in the robustness to be:

$$m(h) = \int_0^\infty \left[(1-h)^+ \widetilde{f}(c_1,\rho) - c_1\right] n(c_1)\, dc_1 \tag{344}$$

$$= (1-h)^+ \widetilde{\Gamma}_2(\rho) - \Gamma_1 \tag{345}$$

where $\widetilde{\Gamma}_2(\rho)$ is the putative value of the total consumption in the 2nd time interval, and it depends on the reference consumption, ρ.

§ The performance requirement is $m(h) \geq \varepsilon$, where $\varepsilon > 0$, namely:

$$(1-h)^+ \widetilde{\Gamma}_2(\rho) - \Gamma_1 \geq \varepsilon \tag{346}$$

§ Solving for h in eq.(346) at equality yields the robustness:

$$\frac{\Gamma_1 + \varepsilon}{\widetilde{\Gamma}_2} = (1-h)^+ \quad \Longrightarrow \quad \boxed{\widehat{h}(\varepsilon,\rho) = \begin{cases} 1 - \frac{\Gamma_1+\varepsilon}{\widetilde{\Gamma}_2(\rho)} & \text{if } \varepsilon \leq \widetilde{\Gamma}_2(\rho) - \Gamma_1 \\ 0 & \text{else} \end{cases}} \tag{347}$$

§ The robustness function in eq.(347) is shown schematically in fig. 49, demonstrating the properties of trade off and zeroing.

§ Fig. 50 shows robustness curves for two different values of the reference consumption, demonstrating that their robustness curves will not cross if their putative total consumptions are different.

§ Summarizing fig. 50, we see that a change in the reference consumption, ρ, that causes a **decrease** in total putative consumption, $\widetilde{\Gamma}_2(\rho_2) < \widetilde{\Gamma}_2(\rho_1)$, also causes a **increase** in the cost of robustness: the robustness curve for ρ_2 is steeper than for ρ_1.

§ This is the reverse of what was observed with respect to fig. 46. In both cases, however, there is no curve crossing.

§ Like the case of fig. 46, the previous observation implies a **re-enforcing impact on the robustness** of the two aspects. Lower $\widetilde{\Gamma}_2(\rho_2)$ shifts the robustness curve to the left (not to the right), and makes the ρ_2 robustness curve less steep which raises the cost of robustness. The result is again no crossing of the robustness curves.

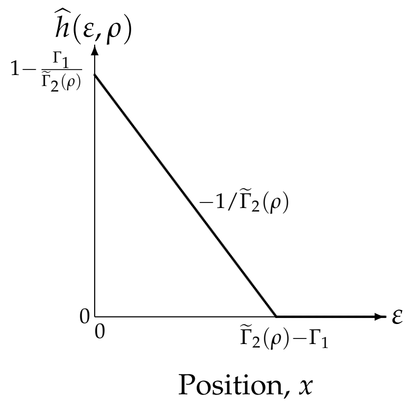

Figure 49: Robustness curve, eq.(347), showing zeroing and trade off.

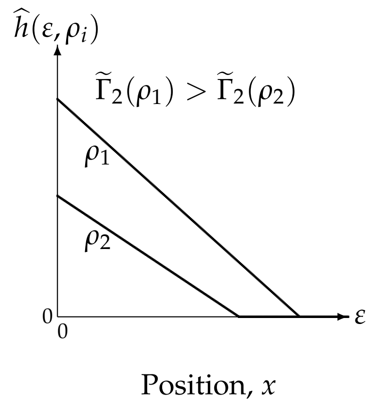

Figure 50: Two robustness curves for different values of the reference consumption.