Midterm Exam, 27.5.2025
Economic Decision Making for Engineers

Derived directly from Problem 38 a, b, c, f

Problems

1. You will take a loan of \$10,000 for 5 years with yearly compound interest of 5%. There is no inflation. You will repay \$2,500 at the end of each of the first 4 years. What is the payment at the end of the 5th year? What is the interest that accrues during the 5th year?

2. You will earn NIS15,000 at the end of each year for 10 years. What is the present worth of this income stream if the interest is 4%. There is no inflation.

3. At the end of each year, for N years, you will earn A and spend C. Both A and C are constant but uncertain with this info-gap model:

$$\mathcal{U}(h) = \left\{ (A, C) : \left| \frac{A - \widetilde{A}}{s_A} \right| \leq h, \left| \frac{C - \widetilde{C}}{s_C} \right| \leq h \right\}, \quad h \geq 0 \tag{1}$$

   where $\widetilde{A}$, $\widetilde{C}$, s_A and s_c are all positive and known. You require that the present worth of this N-year program be no less than $P_{\rm c}$. Derive an explicit algebraic expression for the robustness function.

4. You wish to choose an investment option, and you require that the future worth will be no less than $F_{\rm c}$. You are offered two options between which you can choose. Option 1 guarantees a future worth of exactly F_1. The second option is uncertain, and its robustness function for future worth is:

$$\widehat{h}_2(F_{\rm c}) = 1 - \frac{F_{\rm c}}{2F_1} \tag{2}$$

   or zero if this is negative. Which option would you choose, as a function of $F_{\rm c}$, where your preference is for the more robust option?

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Year	Debt at start of year	Interest accrued in year	Payment at end of year
1	10,000	500	2,500
2	8,000	400	2,500
3	5,900	295	2,500
4	3,695	184.75	2,500
5	1,379.75	**68.9875**	**1448.7375**

Table 1: Solution of problem 1. Currency is \$.

Solutions

(1) We solve this problem one year at a time, as explained in the table 1. The payment in the last year is \$1448.7375. The interest that accrues in the 5th year is \$68.9875.

(2) The present worth of the income stream is:

$$\text{PW} = \sum_{k=1}^{N}(1+i)^{-k}A = \frac{1-(1+i)^{-N}}{i}A \tag{3}$$

With $N = 10$, $i = 0.04$ and $A = 15,000$ we find:

$$\text{PW} = 8.11089 \times 15,000 = \boxed{\text{121,663.44}} \tag{4}$$

(3) The present worth of the cash flow is:

$$\text{PW} = \sum_{k=1}^{N}(1+i)^{-k}(A-C) = \underbrace{\frac{1-(1+i)^{-N}}{i}}_{\delta}(A-C) \tag{5}$$

which defines the discount factor δ. The robustness is defined as:

$$\widehat{h}(P_{\text{c}}) = \max\left\{h:\ \left(\min_{A,C\in\mathcal{U}(h)}(A-C)\delta\right) \ge P_{\text{c}}\right\} \tag{6}$$

Let $m(h)$ denote the inner minimum, which is the inverse of the robustness function. This inner minimum occurs for $A = \widetilde{A} - s_A h$ and $C = \widetilde{C} + s_C h$. Thus:

$$m(h) = \left[\widetilde{A} - \widetilde{C} - h(s_A + s_C)\right]\delta \ \ge\ P_{\text{c}} \quad \Longrightarrow \quad \boxed{\widehat{h}(P_{\text{c}}) = \frac{(\widetilde{A}-\widetilde{C})\delta - P_{\text{c}}}{(s_A + s_C)\delta}} \tag{7}$$

or zero if this is negative.

(4) Option 1 guarantees a future worth of exactly F_1. This can be expressed as the following robustness function:

$$\widehat{h}_1(F_{\text{c}}) = \begin{cases} \infty & \text{if } F_{\text{c}} \le F_1 \\ 0 & \text{else} \end{cases} \tag{8}$$

Comparing $\widehat{h}_1(F_{\text{c}})$ with $\widehat{h}_2(F_{\text{c}})$ in eq.(2), p.1, we see:

$$\widehat{h}_2(F_{\text{c}}) \ > \ \widehat{h}_1(F_{\text{c}}) \quad \text{if} \quad F_1 < F_{\text{c}} < 2F_1 \tag{9}$$

$$\widehat{h}_2(F_{\text{c}}) \ < \ \widehat{h}_1(F_{\text{c}}) \quad \text{if} \quad F_{\text{c}} \le F_1 \tag{10}$$

$$\widehat{h}_2(F_{\text{c}}) \ = \ \widehat{h}_1(F_{\text{c}}) \quad \text{if} \quad F_{\text{c}} \ge 2F_1 \tag{11}$$

Thus, based on a robust preference ranking, we prefer option 2 if $F_1 < F_{\text{c}} < 2F_1$. We prefer option 1 if $F_{\text{c}} \le F_1$. We are indifferent otherwise.

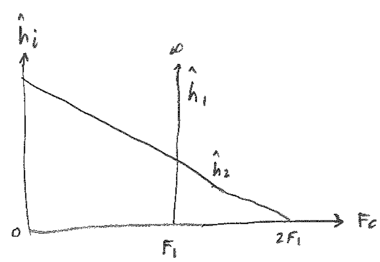

Figure 1: Robustness curves for problem 4.