21. **Exchange rate devaluation.** (p.88) (DeGarmo, 9-30, p.400) A US firm requires a 26% rate of return in US$ on an N-year investment in a foreign country. The real return in the foreign currency in year k is $R_{r,\mathrm{for}}$. The year-0 exchange rate is $r_0 = 1$. The initial investment is S US$. There is no inflation in either country.

    (a) If the currency of the foreign country is expected to devalue at an average annual rate of 8% with respect to the US$, what rate of return in the foreign country would be required to meet the firm's requirement?

    (b) If the dollar is expected to devalue at an average annual rate of 6% with respect to the currency of the foreign country, what rate of return in the foreign country would be required to meet the firm's requirement?

29. **Future foreign earnings.** (based on exam, 21.7.2014) (p.100) Your employment contract states that at the end of k years from now you will receive a payment, in \$'s, of the fixed sum A_k. The general price inflation of \$'s will be f_j in year j for $j = 1, \ldots, k$. The exchange rate between \$'s and pesos at the end of year k will be r_k peso/\$. The general price inflation of pesos will be ϕ_j in year j for $j = 1, \ldots, k$.
    (a) What is the real value of A_k in \$'s at the start of year 1?
    (b) What is the real value of A_k in pesos at the start of year 1 if you transfer the payment when it is received, which is at the end of year k?
    (c) The value of the payment A_k is firmly fixed by contract. However, suppose the dollar inflation rates f_j are highly uncertain, while the peso inflation rates are very stable and well known. The exchange rate r_k is also highly uncertain. Consider the following info-gap model for f_j and r_k:

$$\mathcal{U}(h) = \left\{ r_k, f_j, j = 1, \ldots, k : \ r_k \geq 0, \ \left| \frac{r_k - \widetilde{r}_k}{\widetilde{r}_k} \right| \leq h, \ \left| \frac{f_j - \widetilde{f}_j}{\widetilde{f}_j} \right| \leq h \right\}, \quad h \geq 0 \qquad (13)$$

    You require that the real peso value, at the start of year 1, of the year-k earnings in \$'s, be no less than $R_{\rm c}$. Derive an explicit algebraic expression for the robustness to uncertainty.

Solution to Problem 21, Exchange rate devaluation (p.18). An initial US$ investment S has returns $R_{k,\text{dom}}$ in US$ in years $k = 1, \ldots, N$, or returns $R_{k,\text{for}}$ in the foreign currency, where:

$$R_{k,\text{dom}} = r_k R_{k,\text{for}} \tag{277}$$

and:

$$r_k = (1+\varepsilon)^{-k} r_0 \tag{278}$$

Recall that $r_0 = 1$ US$ per unit of foreign currency. For (a) $\varepsilon = 0.08$, and for (b) $\varepsilon = -0.06$.

The PW_{dom}, calculated with domestic currency, is:

$$\text{PW}_{\text{dom}} = \sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} R_{k,\text{dom}} \tag{279}$$

where $i_{\text{dom}} = 0.26$.

The PW_{for}, calculated with foreign currency, is:

$$\text{PW}_{\text{for}} = \sum_{k=1}^{N} (1+i_{\text{for}})^{-k} R_{k,\text{for}} \tag{280}$$

where i_{for} must be determined.

[Note: The PW terms in eqs.(279) and (280) do not include the initial investment. If one includes them then they become:

$$\text{PW}_{\text{dom}} = -S + \sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} R_{k,\text{dom}} \tag{281}$$

$$\text{PW}_{\text{for}} = -r_0 S + \sum_{k=1}^{N} (1+i_{\text{for}})^{-k} R_{k,\text{for}} \tag{282}$$

Recall that $r_0 = 1$. Thus the same "$-S$" terms on both sides of eq.(283) cancel out.]

The PW's in eqs.(279) and (280) must be equal, after exchanging one of the currencies. Thus, using eqs.(277) and (278):

$$\sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} (1+\varepsilon)^{-k} R_{k,\text{for}} = \sum_{k=1}^{N} (1+i_{\text{for}})^{-k} R_{k,\text{for}} \tag{283}$$

This relation holds if:

$$(1+i_{\text{dom}})^{-k} (1+\varepsilon)^{-k} = (1+i_{\text{for}})^{-k} \quad \text{for all } k \tag{284}$$

Thus:

$$i_{\text{for}} = (1+i_{\text{dom}})(1+\varepsilon) - 1 \tag{285}$$

For (a):

$$i_{\text{for}} = (1+0.26)(1+0.08) - 1 = 0.3608 \tag{286}$$

For (b):

$$i_{\text{for}} = (1+0.26)(1-0.06) - 1 = 0.1844 \tag{287}$$

Solution to Problem 29, Future foreign earnings, (p.25).

(29a) The real value at time 0, in \$'s, of the \$ payment A_k at the end of year k is, from eq.(272) (see the derivation there, p.86):

$$R_{0,k} = A_{0,k} = A_k \prod_{j=1}^{k} (1 + f_j)^{-1} \tag{369}$$

(29b) The foreign nominal value in year k is:

$$A_{k,\text{for}} = r_k A_k \tag{370}$$

Eq.(369) applies to pesos when using the peso inflation rates, so the real value at time 0, in peso's, of the \$ payment A_k after transfering to pesos at the end of year k is:

$$R_{0,k,\text{for}} = A_{0,k,\text{for}} = A_{k,\text{for}} \prod_{j=1}^{k} (1 + \phi_j)^{-1} \tag{371}$$

$$= r_k A_k \prod_{j=1}^{k} (1 + \phi_j)^{-1} \tag{372}$$

(29c) The uncertainty in the \$ inflation is irrelevant, because the \$ inflation rate does not influence the peso value. This is because the value of A_k is fixed in dollars. However, the uncertainty in the exchange rate is relevant. The robustness is:

$$\widehat{h} = \max \left\{ h : \left(\min_{r_k \in \mathcal{U}(h)} R_{0,k,\text{for}}(r_k) \right) \geq R_\text{c} \right\} \tag{373}$$

Let $m(h)$ denote the inner minimum, which occurs for the lowest exchange rate at horizon of uncertainty h: $r_k = (1-h)^+ \widetilde{r}_k$. From eq.(372):

$$m(h) = (1-h)^+ R_{0,k,\text{for}}(\widetilde{r}_k) \tag{374}$$

Equating this to R_c and solving for h yields the robustness:

$$\widehat{h} = 1 - \frac{R_\text{c}}{R_{0,k,\text{for}}(\widetilde{r}_k)} \tag{375}$$

or zero if this is negative.